Multistatic passive radar performance in terms of positioning accuracy is largely dependent on the geometry, and can be optimised by: (i) selecting the most appropriate transmitters of opportunity among the available ones, and (ii) identifying the optimal receiver location. These steps are clearly not independent so that a joint optimization is required. Last year, I have introduced a procedure to obtain a multistatic system which is effective for the surveillance of aircrafts flying a specific assigned trajectory, as it is, for example, the case of an approach path or a portion of an air route. The optimization procedure introduced in my first year of research was obtained by using a simplifying hypothesis on the range accuracy, that was set coincident with the range resolution and independent of the actual SNR (Signal to Noise Ratio). The resulting performance parameter, represented by the CRLB, depends on the geometry only in terms of relative angular values. In contrast, the SNR affects largely both detection probability and range accuracy and, in the specific case of a passive radar, it rapidly changes from a very high value, very close to the transmitter, to low values, causing a drop of the probability of detection up to values between 0.8 and 0.6. Therefore, it was clear that for passive radar performance assessment, the SNR value must be carefully included into the evaluation. This is done by both (a) considering a range accuracy dependent on the SNR in the evaluation of the CRLB, (b) and by removing the assumption of $P_d=1$, intrinsically made when evaluating the standard CRLB.

The CRLB plays an important role in the estimation theory, because it theoretically predicts the best achievable second-order estimation error performance; however, when operating after a detection test, it must be modified accordingly. In [1], the theoretical CRLB for $P_d < 1$ and $P_{fa}=0$ (i.e. absence of false alarms) has been calculated via the enumeration of all possible sequences of detections and miss detections, given a certain scan number, for a conventional active radar. In my work of this year, I provided an extension of this approach to a multistatic passive radar where, for each bistatic TX-RX couple, an independent detection test is applied, so that there are multiple independent decisions for each set of measurements.

I proceeded as follows. At first I derived the CRLB with uncertain measurements for the multistatic passive radar, by extending the enumeration approach to the multisensor case, obtaining a realistic evaluation of the positioning accuracy that fully includes the effect of SNR. Then, I exploited the derived CRLB to refine the multistatic passive radar optimization scheme. In particular, after describing the constraints for the relative position of transmitters and receivers, due to the antennas radiation pattern and signal processing, I made extensive simulations and obtained specific results.

To extend the approach in [1-3] to the case of $N$ sensors, with detection probability $P_d < 1$, $i=1,...,N$ and performing an independent detection test, I proceeded as follows.

I assumed each sensor performs $K$ bistatic range measurements: $R_{Bi,k}=R_i,k(x,y)+\varepsilon R_i,k$, with $k=1,...,K$, that are assumed independent and with Gaussian probability density function with expected value $E(R_{Bi,k})=R_i,k(x,y)$ and variance $E((R_{Bi,k}-E(R_{Bi,k}))^2)=\sigma_i^2\varepsilon_i,k$. Next, I introduced an observable boolean variable $d_{i,k}$ that corresponds to the event that the target was detected or missed at time $k$:

$$d_{i,k} = \begin{cases} 
1, & \text{if the sensor } i \text{ detects the target at time } k \\
0, & \text{if the sensor } i \text{ misses the target at time } k 
\end{cases}$$ (1)

The $i$-th sensor can form $2K$ possible detection/miss sequences:
The number of detections in the specific sequence \( m \), for a specific sensor \( i \), is:

\[
\Delta_{i,K,m} = \sum_{k=1}^{K} d_{i,k}^{(m)}
\]

(2)

The number of misses in \( S_{i,k}^{(m)} \):

\[
\bar{\Delta}_{i,K,m} = K - \Delta_{i,K,m}
\]

(3)

The probability of occurrence of a particular detection/miss sequence at the sensor \( i \), is:

\[
\Pr\{S_{i,K}^{(m)}\} = P_{d,i}^{(m)} \cdot (1 - P_{d,i})^{K-m}
\]

(4)

If we consider \( N \) bistatic couples, we will have \( L=2K \) possible detection/miss sequences and the probability of occurrence of the \( l \)-th sequence will be:

\[
\Pr\{S_{i,K}^{(l)}\} = \prod_{i=1}^{N} \Pr\{S_{i,K}^{(m)}\}
\]

(5)

The CRLB is then the average over the scenario dependent values. The weighted mean of all the possible covariance matrix, where the weight will be the probability of occurrence of the particular detection/miss sequence:

\[
J_{j,h} = \sum_{i=1}^{L} \Pr\{S_{i,K}^{(l)}\} \sum_{i=1}^{K} \sum_{k=1}^{K} \frac{1}{\sigma_{i,k}^2} \frac{\partial R_{B_i,k}}{\partial \Theta_j} \frac{\partial R_{B_i,k}}{\partial \Theta_h}
\]

(6)

where \( \Theta \) is the vector of the parameters of interest: \( \Theta=[x,y] \).

The CRLB was used to assess the ideal accuracy of the obtained target position estimation, therefore, the lower bound for the estimation variance of the two parameters of interest is:

\[
E\{\Theta_j - \hat{\Theta}_j\}^2 = \sigma_{\Theta_j}^2 = \{J^{-1}\}_{jj}
\]

(7)

(8)

recalling that \( \sigma_x^2 = \{J^{-1}\}_{11} \) and \( \sigma_y^2 = \{J^{-1}\}_{22} \). Based on this CRLB I evaluated the performance of a passive multistatic radar system by means of the 2D horizontal accuracy:

\[
\sigma_H = (\sigma_x^2 + \sigma_y^2)^{1/2}
\]

where \( \sigma_x^2 \) and \( \sigma_y^2 \), the lower bound for the estimation variance of the two parameters of interest, are calculated using the extensive enumeration CRLB.

**Constraints for Receiver Displacement**

Briefly recalling the constraints, introduced last year, for the relative positioning of transmitters, receivers, and surveillance region with specific reference to the passive radar sensor in order to obtain a correct configuration it is required that: (i) the whole target’s trajectory is illuminated from the main beam of the receiver antenna; (ii) both the direct signals from the two transmitters arrive in the back lobe region of the receiving antenna (so that the direct signal is attenuated); (iii) the surveillance region does not include regions from which the target echo is received with zero-Doppler (so that the target echo is not filtered away by the clutter cancellation filter).

The three requirements must be jointly verified assuming that the passive radar has to operate with a single receiver. Allowing more receivers would provide greater flexibility but at higher cost and will be object of future research.

**Case of Study and Its Geometry**

In my case study I assumed that 17 transmitters shown in Fig.1 are available and considered the surveillance of a target that is expected to move on the red line from point A to B.
A subset of them (two in my case study) must be selected for the multistatic passive radar. While the transmitters are assumed to use an omni-directional antenna, the receiver uses a directional antenna pattern that is characterized by: a maximum gain $G_t$, a -3dB beamwidth $\alpha=90^\circ$, a low level back-lobes angular region $\gamma=180^\circ$ wide (gain lower than $G_t-X$ dB, with X from 12 to 30 depending on carrier frequency), and two intermediate angular regions $\beta=45^\circ$ wide with gain values between $G_t-X$ dB and $G_t-3$ dB. I considered each of the 136 transmitters couples obtainable with the available transmitters and assumed that each bistatic couple performs two measures of bistatic range ($K=2$) for each target position (A, M, B), and that the target movement is negligible. Thus, the element $(j,h)$ of the Fisher Information Matrix is given by:

$$J_{j,h} = \sum_{l=1}^{136} \Pr\{S^{(l)}\} \sum_{i=1}^{2} \frac{1}{\sigma_{\varepsilon_i}^2} \frac{\partial R_{\theta_{i,j}}}{\partial \Theta_{i,j}} \frac{\partial R_{\theta_{h,j}}}{\partial \Theta_{h,j}} \sum_{k=1}^{2} d_{i,k}^{(l)}$$

For each couple, and for each target position, I drew a map of the maximum $\sigma_H$ on the whole trajectory using the CRLB above, then selected the best receiver position as the one that minimizes this value.

I analysed and compared four sub-cases: in case (A) it is assumed $P_d=1$ and the measurement accuracy equal to the bistatic range resolution for all measurements, in case (B) $P_d=1$ and measurement accuracy depending on SNR; in case (C) $P_d$ depends on SNR and measurement accuracy is constant; case (D) both $P_d$ and measurement accuracy depend on SNR.

A. $P_d=1$ and $\sigma_\varepsilon=2000m$

I did not take into account the uncertainty of the detections (namely I assumed $P_d=1$) and a measurement accuracy of $\sigma_\varepsilon R=2000m$ is considered (assumed comparable to the bistatic range resolution of FM radio).

B. $P_d=1$ and $\sigma_\varepsilon=f(SNR)$

I assumed there are not missed detection (namely again $P_d=1$) and introduced the dependence of the measurements accuracy on the Signal to Noise Ratio:

$$\sigma_\varepsilon = \frac{c}{B_i \sqrt{2SNR_i}}$$

were the SNR is evaluated in the following way:

$$SNR_i = \frac{P_t G_t G_i \sigma_\varepsilon^2 \tau_i}{(4\pi)^3 kT_{\text{abs}} R_t^2 R_{\varepsilon_i}^2 N_i L_{\text{avg}}}$$

and the considered parameters are: transmitters power $P_t=0.5$ kW, Radar Cross Section $RCS=1$ m$^2$, receiver bandwidth $B=200$ kHz, receiver noise figure $NF=10$ dB, wavelength $\lambda=3$ m, integration time $t_i=1$ s, propagation loss $L=10$ dB, transmitter antenna gain $G_t=0$ dB, receiver antenna gain $Gr=10$ dB.

C. $P_d=f(SNR)$ and $\sigma_\varepsilon=2000m$
I assumed that the accuracy of each measurement is \( \sigma_R=2000\text{m} \) and considered the detection probability given by the following expression (Swerling I target model):

\[
P_{di} = P_{fa} \frac{1}{1+\text{SNR}},
\]

where \( P_{fa}=10^{-4} \) and the value of SNR in eq. (11).

**D. \( P_d= f(\text{SNR}) \) and \( \sigma_e= f(\text{SNR}) \)**

In this case I considered both the dependence on the SNR as in cases B and C, namely in eq. (10)-(12), with the same values assumed above.

**RECEIVER POSITIONING AND ITS RESULTS**

In this section I aimed at jointly selecting the best couple of transmitters, among the available ones, and the best receiver position to localize the target along the assigned trajectory with the highest accuracy for each one of the considered air traffic control study cases.

In order to perform a correct receiver placement I had to consider the geometrical constraints previously described.

Fig. 2 shows the admissible receiver positions (white areas) for a specific transmitters couple (TX3-TX5) for each one of the constraints listed in Section III.

The following comments are in order: (a) to illuminate the whole target trajectory with the main beam of the receiving antenna the receiver cannot be too close to the target; (b) in order to receive the direct signal from TX3 in the back-lobe region, the admissible area is close to the points of the target trajectory since, at the receiver position, the angular distance between each target position and the transmitter position has to be always higher than \( \beta \). The admissible region for TX5, is equal to the one of TX3 but is symmetrical with respect to the target trajectory. (c) The zero Doppler frequency constraint gave the same admissible regions for TX3 and TX5; (d) only two admissible regions are identified, a bigger one above the point A and a smaller region in the proximity of point B. This final admissible region has been obtained as the intersection of all the constraints for both transmitters.

![Fig. 2 Admissible receiver positions for: (a) Main beam steering constraint (\( \alpha=90^\circ \)); (b) Direct signals in the back-lobe region constraint for TX3 (\( \beta=45^\circ \)); (c) Zero-Doppler frequency constraint for TX3 and TX5; (d) Admissible region](image)

The performed analysis showed that the transmitters couples with the largest admissible regions are composed by transmitters on the external circle in Fig. 1; however, they obviously do not provide good results in terms of SNR and consequently in terms of measurement accuracy and detection probability. To compare the performance obtained using different couples of transmitters, I considered the best receiver position for each one of them, selected as to maximize the positioning accuracy on the whole target trajectory.
Figs. 3-6 report the horizontal accuracy $\sigma_H$ as a function of the receiver position for the couple TX3-TX5 and the target at point “M” for the four cases considered: A., B., C. and D., respectively. The optimal receiver position, namely the one that minimizes the maximum $\sigma_H$ on the whole trajectory, is reported in the figures with a green ‘*’ symbol.

Comparing the results of Figs. 3-6, I noticed that in case A (Fig 3), where it is only considered the effect of the system geometry on the horizontal accuracy, performance are limited, because the measurement accuracy is considered equal to the range resolution of the system. Moreover, the analysis showed that, taking into account just the effect of the system geometry, other transmitters couples allow to achieve better results. In Fig 4 (case B) it is stressed the positive effect of the SNR on the measurements accuracy, especially when the receiver is in the proximity of the target positions. Fig. 5 shows the negative effect of SNR on the detection probability with respect to the case A, where the $P_d$ is assumed equal to one. In Fig 6. (case D) I took into account the effect of the SNR, both on the measurement accuracy and on the detection probability as described in eq. (10)-(12); I also found a slight worsening of the horizontal accuracy, much faster than in case B (Fig.4), as the distance of the receiver from the target increases; although, it is not clearly observable from the figure.

The transmitter couple TX3-TX5 is one of the best couple for cases B., C. and D, since the transmitters are near to the target trajectory and, consequently, both the detection probability and the measurement accuracy are good. The study showed that the difference between results obtained in case B and case D is much significant when the transmitters are far from the target trajectory, because the detection probability is lower. So, in order to perform a correct design of a multistatic passive radar, it is important to take into account the detection probability.

Notice that the above optimization of the receiver position has been performed without considering the constraints .When these constraints are also considered, the optimum receiver position is likely to be modified. As an example in fig.3-6 a green ‘●’ symbol is used to indicate the best receiver
position subject to the geometrical constraints. As it is clearly observable, in each one of the four cases, the best receiver position without constraints is not admissible if the constraints previously recalled (see Fig. 2 d) are considered; thus it is necessary tolerate a deterioration of the horizontal accuracy.

The increase of the estimation error, due to the introduction of the geometrical constraints, is also observed in Table I, which reports the comparative performance of a few selected transmitters couples among the best ones. Specifically, Table I shows the value of the maximum localization error obtained with the best receiver position along the whole target trajectory for each one of the four considered cases (A., B., C. and D.). The second column refers to the case without geometrical constraints, while the third column reports the results obtained when the receiver position is modified to match the constraints.

As it is clearly observable, when the constraints are considered for a given transmitters couple, the localization error (namely the lowest value for the maximum horizontal accuracy along the target trajectory) might be significantly degraded. Moreover, notice that some of the transmitters couples, providing an overall good accuracy, are excluded by the constraints from the set of possible transmitters couples, since the admissible region for the receiver placement is empty.

The analysis showed that the set of transmitters couples unusable for a multistatic passive radar are the following ones: 1-4; 1-12 (4-8); 4-16; 8-12; 12-16; 4-7 (4-9); 4-15 (4-17).

<table>
<thead>
<tr>
<th>Case</th>
<th>Best couples</th>
<th>Maximum Error without constraints</th>
<th>Maximum Error with constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>12-15 (12-17)</td>
<td>1308 m</td>
<td>1331 m</td>
</tr>
<tr>
<td></td>
<td>4-15 (4-17);</td>
<td>1308 m</td>
<td>Not Allowed</td>
</tr>
<tr>
<td></td>
<td>7-12 (9-12);</td>
<td>1316 m</td>
<td>1370 m</td>
</tr>
<tr>
<td></td>
<td>4-7 (4-9);</td>
<td>1316 m</td>
<td>Not Allowed</td>
</tr>
<tr>
<td>B</td>
<td>1-4</td>
<td>14.2 m</td>
<td>Not Allowed</td>
</tr>
<tr>
<td></td>
<td>3-5</td>
<td>20.2 m</td>
<td>33.8 m</td>
</tr>
<tr>
<td>C</td>
<td>3-5</td>
<td>1455.8 m</td>
<td>1707.5 m</td>
</tr>
<tr>
<td></td>
<td>1-4</td>
<td>1487.4 m</td>
<td>Not Allowed</td>
</tr>
<tr>
<td>D</td>
<td>1-4</td>
<td>14.2 m</td>
<td>Not Allowed</td>
</tr>
<tr>
<td></td>
<td>3-5</td>
<td>20.3 m</td>
<td>34.1 m</td>
</tr>
</tbody>
</table>

It is also remarkable that for many transmitters couples the admissible region for the receiver is very limited, so that it appears very inefficient to evaluate the CRLB over the whole plane, and to exclude a large region afterward in order to select the best receiver position. However, in the above analysis I considered the whole region in order to better understand the role of the different parameters.

Comparing the four cases introduced, I observed that the best results in terms of localization accuracy are obtained in case B, thanks to the benefit of the Signal to Noise Ratio on the measurements accuracy. The worst case is case C, because the results are only affected by a detection probability lower than one. The most realistic results are obtained for case D in which both the effects of the SNR and of the $P_d$ are taken into account.

CONCLUSIONS

During my study I derived a version of the CRLB for the localization of a target by a multistatic passive radar (multisensory case), based on the extensive enumeration approach to include the effect of a sensor probability of detection lower than unity. A thorough analysis of constraints for passive radar operation, combined together with the Extensive Enumeration CRLB for the location accuracy, lead to an effective optimization procedure for a multistatic passive radar, operating with two transmitters and a single receiver.

The enumeration method is accurate but computationally expensive. The theoretical formula involves the evaluation of exponentially growing number of possible miss/detection sequences.
Its application to target tracking using a multistatic passive radar system is currently the
continuation of my research.
The same analysis, based on the extensive enumeration method has been performed considering a
real scenario, considering its topography, and introducing the further constraints of the visibility
between the transmitters and the receiver, and between the receiver and the target.
This new constraints significantly affects the design of the system, but of course it is essential to
take it into account in order to deploy a passive multistatic radar.
In the last steps of my research I have introduced the problem of the surveillance of a Area of
Interest and from this point I foresee to start my third year of research in order to derive a
systematic design method for a net of passive sensors exploiting the existing illuminators of
opportunity for the monitoring and the surveillance of the airspace.

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