Optimized Signal Shaping for Multi-Antenna Networks impaired by Multiple Access Interference

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Abstract

This paper deals with the optimized signal shaping for Multiple-Input Multiple-Output (MIMO) systems to maximize information throughput conveyed by pilot-based Multi-Antenna systems affected by both spatially colored Multiple Access Interference (MAI) and errors in the available channel estimates. The architecture of the Minimum Mean Square Error (MMSE) MIMO channel estimator is derived and the related analytical conditions for the optimal design of space-time training sequences are provided. Afterward, closed form expressions for the maximum information throughput sustained by the considered system for Gaussian distributed input signals are given and, then, a novel power allocation algorithm for the asymptotical achievement of the system capacity is developed. Results of several numerical tests and performance comparisons are also presented supporting actual effectiveness of the developed analytical tool for Multi-Antenna networks working in an uncoordinated "ad-hoc" mode. Finally, some considerations about novel effective Multiple Access strategies exploiting Space-Division are pointed out.

Index Terms

- Multi-Antenna, MAI, imperfect channel estimation, information throughput, ad-hoc network, power management, signal-shaping, space-division multiple-access.

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I. INTRODUCTION AND GOALS

Due to fast increasing demand lastly experienced of pervasive high-throughput Personal Communication Services (PCSs) based on small-size power-saving palmtops [8], the requirement for "always on" mobile data access based on uncoordinated "ad-hoc" type networking architecture is expected to dramatically increase within few next years [18,20]. To account for the resulting demand for large channel capacity, the spatial dimension provided by wireless communication systems may be adequately exploited. As a consequence, in these last years an increasing attention has been paid to the development of array-equipped transceivers for wireless PCSs [25].

A. Related Works

In this respect, all contributions currently published focus on the (ideal) case of transceivers equipped with perfect estimate of the underlying MIMO channel. Specifically, in [1,2] the capacity of MIMO transceivers equipped with perfect channel estimate at both transmit and receiver units is analyzed, while in [23] the case of perfect channel estimate at the receiver is examined when the number of transmit/receiver antennas grows. Although above mentioned assumption of perfect channel estimates may be considered reasonable in quasi-static application scenarios (as those encountered, for example, in Wireless Local Loop applications [1]), nevertheless this last is expected to fall short in emerging applications for high-quality PCSs [9,10]. Furthermore, recently some works deal with imperfect channel estimation assumption [...] but no works in literature seem to address the problem of channel estimation in the presence of spatially colored Multiple Access Interference (MAI).

B. Proposed Contributions

Therefore, in this work we focus on the ultimate information throughput conveyed by pilot-based wireless MIMO systems equipped with imperfect channel estimate at both transmit and receiver and impaired by spatially colored MAI. Thus, the result we present may be viewed as a generalization of those published in [1,2] and, de facto, comprehends them as a limit case. Specifically, main contributions of this work may be summarized as follows. First, we develop the optimal MMSE channel estimator for pilot-based MIMO systems impaired by spatially-colored MAI. Second, we provide the analytical properties characterizing the optimized space-time training sequences and then we show as these last are related to the statistics of the spatial MAI. Third, we develop closed-form analytical expressions for computing the information throughput sustained by the considered MIMO systems for the case of Gaussian distributed input signals and then we point out several asymptotical operating conditions.
guaranteeing the achievement of the corresponding system capacity. Fourth, we propose an iterative algorithm for the optimized power allocation and signal-shaping when imperfect channel estimate are available at both transmit and receiver. So doing, we generalize the water-filling approach previously reported in [1,2]. Finally, we produce several numerical results and performance comparisons supporting the effectiveness of the proposed power allocation algorithm in networking scenarios operating in "ad-hoc" mode. On the basis of these last, some (novel) guidelines about the optimized design of Multiple-Access strategies exploiting space-division are pointed out.

C. Organization of the work

The remainder of the paper is organized as follows. After the system modeling of Sect.II, the optimal MMSE MIMO channel estimator is developed in Sect.III. Next Sect.IV deals with both the evaluation of the conveyed information throughput and related optimized power allocation over transmit antennas. After a short modeling in Sect.V of the spatial MAI arising in Multi-Antenna "ad-hoc" networks, Sect.VI presents several numerical plots and performance comparisons testing the effectiveness of the presented analytical framework. Final Sect.VII points out some general guidelines about the overall design of MAI-impaired Multi-Antenna pilot-trained transceivers. Before proceeding, few words about the adopted notation. Capital letters indicate matrices, lower-case underlined symbols denote vectors while characters overlined by arrow \( \rightarrow \) denote block-matrices and block-vectors. Apexes \( * \), \( ^T \), \( ^\dagger \) mean conjugation, transposition and conjugate-transposition respectively, while lower-case letters will be used for scalar quantities. In addition, \( \det[A] \) and \( \text{Tr}A \) mean determinant and trace of matrix \( A \equiv [a_1 \ldots a_m] \), while \( \text{vect}(A) \) indicates the (block) vector obtained by the ordered stacking of the columns of \( A \). Finally, \( I_m \) is the \((m \times m)\) identity matrix, \( ||A||_F \) is the Euclidean norm of the matrix \( A \), \( A \otimes B \) is the Kronecker product of the matrix \( A \) by matrix \( B \), \( 0_m \) is the m-dimensional zero-vector, \( \lg \) denotes natural logarithm and \( \delta(m,n) \) is the Kroenecker delta.

II. The System Modeling

The application scenario considered in this work is that of emerging local wireless "ad-hoc" networks [18,20] where a (large) number of uncoordinated transmit-receive nodes simultaneously communicate over a limited-size hot-spot cell and then give arise to multiple access interference (MAI) [18]. The (complex base-band equivalent) point-to-point radio channel linking a transmitter node Tx to the corresponding receiving node Rx is sketched in Fig.1. Simply stated, it is composed by a transmit unit equipped with \( t \geq 1 \) antennas communicating to a receive unit equipped with \( r \geq 1 \) antennas via a MIMO radio channel...
impaired by both slow-variant flat Rayleigh fading\(^1\) and additive multiple access interference induced by adjacent nodes active over the same hot-spot cell. The path gain \(h_{ji}\) from transmit antenna \(i\) to receive one \(j\) may be modelled as a complex zero-mean unit-variance proper complex random variable (r.v.) \([5,6,7,8]\) and, for sufficiently spaced apart antennas, overall path gains \(\{h_{ji} \in \mathbb{C}, \ 1 \leq j \leq r, \ 1 \leq i \leq t\}\) may be considered mutually uncorrelated\(^2\). Furthermore, for low-mobility applications as those serving users nomadic over hot-spot cells, the path gains \(\{h_{ji}\}\) may be also assumed time-invariant over \(T \geq 1\) signalling periods, after which they change to new statistically independent values held for another \(T\) signalling periods and so on. The resulting “block-fading” model well captures the main features of several frequency-hopping or packet based interleaved 4G systems, where each transmitted packet is detected independently of any other \([7,18,19]\). About the MAI affecting the link of Fig.1, its statistics mainly depend on the network topology \([1,2,20]\), so that in the application scenario here considered it is reasonable to assume these last constant over (at least) an overall packet. However, since both path gains \(\{h_{ji}\}\) and MAI statistics may change from a packet to another, we consider that Tx and Rx in Fig.1 are not aware of them at the beginning of each transmitted packet. Hence, according to Fig.2 we assume that the coded and modulated streams radiated by transmit antennas are split into packets composed by \(T \geq 1\) slots, where the first \(T_L \geq 0\) slots are used by Rx for learning the MAI statistics (see Sect.II.A), the second \(T_{tr} \geq 0\) slots are employed for estimating the path gains \(\{h_{ji}\}\) of the forward MIMO channel (see Sect.II.B) and, finally, the last \(T_{pay} \triangleq T - T_{tr} - T_L\) slots convey payload data (see Sect.II.C). Thus, after indicating as \(R_C\) (nats/slot) the information rate of the employed space-time encoder, the spectral efficiency \(\eta\) (nats/sec/Hz) of the described system equates
\[
\eta = \frac{T_{pay}}{T} \cdot \frac{R_C}{\Delta_s B_w},
\]
where \(\Delta_s\) (sec.) and \(B_w\) (Hz) are the slot duration and RF bandwidth of radiated signals.

A. The Learning Phase

During the learning phase (see Fig.2), no signals are radiated by the \(T_x\) transmitter of Fig.1, so to allow the corresponding \(R_x\) receiver to learn the statistics of the impairing MAI. More in particular, the

\(^1\)The assumption of flat fading is met when RF bandwidth \(B_w\) of the signal radiated by each transmit antenna does not exceed the coherence bandwidth \(B_c\) of the MIMO forward channel of Fig.1. This is the operating conditions of the emerging 4G Local Radio Systems (4GLRS) adopting OFDM modulation formats \([8,11,19]\).

\(^2\)For hot-spot local area applications, antenna spacing of the order of \(\lambda/2\) suffices for meeting the above assumption \([15]\). Furthermore, several measures and analytical contributions indicate that the throughput loss induced by the path correlation is (very) limited even for correlation coefficients as high as 0.5-0.6 \([4\text{ and references therein}].\)
r-dimensional (complex column) vector \( \dot{y}(n) \triangleq [y_1(n)\ldots y_r(n)] \) collecting the outputs of the \( r \) receive antennas over the \( n \)-th slot of the learning phase may be modeled as

\[
\dot{y}(n) \triangleq \ddot{d}(n) \equiv \dot{y}(n) + \dot{w}(n), \quad 1 \leq n \leq T,
\]

where the overall disturbance vector \( \ddot{d}(n) \triangleq [\dot{d}_1(n)\ldots \dot{d}_r(n)]^T \) is the superposition of two mutually independent components \( \dot{w}(n) \triangleq [\dot{w}_1(n)\ldots \dot{w}_r(n)]^T \) and \( \dot{v}(n) \triangleq [\dot{v}_1(n)\ldots \dot{v}_r(n)]^T \). The first component accounts for the receiver thermal noise and then \( \{\dot{w}(n) \in \mathbb{C}^r, 1 \leq n \leq T\} \) may be modeled as a zero-mean, proper complex, spatially and temporally white Gaussian sequence with covariance matrix equal to

\[
E\{\dot{w}(n)\dot{w}^T(m)\} = N_0 I_r \delta(m,n),
\]

where \( N_0 \) (watt/Hz) is the level of the receiving thermal noise. Since the second component \( \{\dot{v}(n)\} \) in (2) accounts for the MAI due to multiple co-located transmit nodes active over the same hot-spot cell, it is likelihood to model \( \{\dot{v}(n) \in \mathbb{C}^r\} \) as a zero-mean temporally white, spatially colored proper Gaussian sequence, whose covariance matrix

\[
K_v \triangleq E\{\dot{v}(n)\dot{v}^T(n)\} = \begin{bmatrix}
c_{11} & \cdots & c_{1r} \\
c_{1r} & \cdots & c_{rr} \\
\vdots & \ddots & \vdots \\
c_{r1} & \cdots & c_{rr}
\end{bmatrix},
\]

remains unchanged over time intervals at least equal to the duration of an overall packet\(^3\). However this last may change from a packet to another so that it is reasonable to assume that both \( T_x \) and \( R_x \) nodes in Fig.1 are not aware of the covariance matrix of the overall disturbance \( K_v \). However this last may change from a packet to another so that it is reasonable to assume that both \( T_x \) and \( R_x \) nodes in Fig.1 are not aware of the covariance matrix of the overall disturbance

\[
K_d \triangleq E\{\ddot{d}(n)\ddot{d}^T(n)\} \equiv K_v + N_0 I_r,
\]

at the beginning of each transmitted packet. However, since the received signals \( \{\dot{y}(n)\} \) equate the MAI ones \( \{\dot{d}(n)\} \) during the learning phase, the Law of Large Numbers [26] guarantees that an unbiased

\(^3\)Temporally white assumption implies that \( E\{\dot{v}(n)\dot{v}^T(m)\} \) vanishes for \( n \neq m \) and this assumption is generally well met by interleaved systems adopting FEC Space-Time codes [11]. In addition, argumentations based on the Limit Central Theorem support the Gaussianity assumption, specially for ad-hoc networks composed by a large number of co-located non cooperative active nodes. In this case, since the Gaussian pdf maximizes the differential entropy [12], the Gaussian assumption corresponds to consider a worst-case application scenario. Finally, since the network topology in applications serving nomadic users is slow-variant [20], it is reasonable to consider \( K_v \) in (4) constant at least over the time spanned by the transmission of an overall packet.
and consistent (e.g. asymptotically exact) estimate $\hat{K}_d$ of the (a priori) unknown covariance matrix $K_d$ is given by the following relationship:

$$\hat{K}_d = \frac{1}{T_L} \sum_{n=1}^{T_L} \hat{y}(n)\hat{(y}(n))^\dagger. \quad (6)$$

About accuracy of the estimate in (6), known analytical results (reported, for example, in [3 and references therein]) lead to the conclusion that the relative squared estimation error $||K_d - \hat{K}_d||^2_E/||K_d||^2_E$ vanishes at least as $T_L$, so that we argue that $T_L = 10$ generally suffices to give arise to squared estimation errors below 10%. Thus, since the numerical results reported in Sect.VI.D confirm the (a priori likelihood) conclusion that the throughput loss induced by an imperfect MAI covariance matrix estimate is no substantial in application scenarios of practical interest and, by fact, quasi-vanishes for $T_L$ exceeding 10, thereinafter we assume that at the end of the learning phase (e.g. at step $n = T_L$) the MAI covariance matrix $K_d$ is perfectly estimated at the Rx node, and then it is communicated back to the transmitter via the ideal feedback link of Fig.1.

B. The Training Phase

On the basis of the allowable MAI covariance matrix $K_d$, during the training phase the Tx transmit node of Fig.1 is able to perform the optimized shaping of the deterministic pilot streams $\{\tilde{x}_i(n) \in \mathbb{C}^1, T_L + 1 \leq n \leq T_L + T_{tr}\}, 1 \leq i \leq t$, to be used for estimating the (a priori unknown) $(r \times t)$ path gains $\{h_{ji}\}$ of the MIMO forward channel of Fig.1. In particular, the $\Delta_s$-sampled signals $\{\tilde{y}_{j}(n) \in \mathbb{C}^1, T_L + 1 \leq n \leq T_L + T_{tr}\}, 1 \leq j \leq r$, measured at the output of $j$-th receiving antenna during the training phase may be modelled as

$$\tilde{y}_j(n) = \frac{1}{\sqrt{t}} \sum_{i=1}^{t} h_{ji}\tilde{x}_i(n) + \tilde{d}_j(n), \quad T_L + 1 \leq n \leq T_L + T_{tr}, 1 \leq j \leq r, \quad (7)$$

where the corresponding overall disturbance

$$\tilde{d}_j(n) \triangleq \tilde{v}_j(n) + \tilde{w}_j(n), \quad T_L + 1 \leq n \leq T_L + T_{tr}, 1 \leq j \leq r, \quad (7.1)$$

is independent from path gains $\{h_{ji}\}$ and exhibits the same statistics previously detailed in (4), (5) for

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*We point out that Time-Division-Duplex (TDD) WLANs planned for hot-spot low-mobility applications are typically equipped with (very) reliable duplex channels [15,18] so that in this case the above assumption of an ideal feedback link may be considered well met. In any case, some considerations about the effects of noisy feedback channels will be drawn in the conclusive Sect.VII.
the learning phase. Thus, after assuming the (usual) constraint

\[ \frac{1}{t} \sum_{i=1}^{t} ||\tilde{x}_i(n)||^2 = \tilde{P}, \quad T_L + 1 \leq n \leq T_L + T_{tr}, \quad (8) \]

on the average power \( \tilde{P} \) radiated by the transmit antennas over each slot of the training phase, the corresponding signal to interference-plus-noise ratio (SINR) \( \tilde{\gamma}_j \) measured at the output of j-th receive antenna equates (see eqs. (7), (8))

\[ \tilde{\gamma}_j = \tilde{P} / (N_0 + c_{jj}), \quad 1 \leq j \leq r \quad (8.1) \]

where \( N_0 + c_{jj} \) is the j-th diagonal entry of the MAI matrix \( K_d \) in (5). Therefore, the \( T_{tr} \times r \) (complex) samples gathered at the outputs of the r receive antennas during the overall training phase may be organized into the \( (T_{tr} \times r) \) observed matrix \( \tilde{Y} \equiv [\tilde{y}_1 ... \tilde{y}_r] \) given by [6,7]

\[ \tilde{Y} = \frac{1}{\sqrt{t}} \tilde{X} \mathbf{H} + \tilde{D}, \quad (9) \]

where \( \tilde{X} \equiv [\tilde{x}_1 ... \tilde{x}_t] \) is the \( (T_{tr} \times t) \) matrix of (deterministic) pilot symbols, \( \mathbf{H} \equiv [h_1 ... h_r] \) is the \( (t \times r) \) complex matrix composed by the path gains \{\( h_{ji} \)\} and the \( (T_{tr} \times r) \) matrix \( \tilde{D} \equiv [d_1 ... d_r] \) collects the disturbance samples \{\( \tilde{d}_j(n) \)\} in (7) experienced during the training phase. Obviously, from (8) it follows that the pilot matrix \( \tilde{X} \) in (9) must satisfy the following second order constraint:

\[ \text{Tra}[\tilde{X} \tilde{X}^\dagger] = tT_{tr} \tilde{P}. \quad (9.1) \]

As detailed in Sect.III, the trained observations in (9) are employed by the \( R_x \) receive node of Fig.1 for computing the MMSE matrix estimate \( \hat{\mathbf{H}} \equiv \mathbb{E}\{\mathbf{H} \tilde{Y}\} \) of the MIMO channel \( \mathbf{H} \). In turn, at step \( n = T_L + T_{tr} \) (e.g., at the end of the training phase) this estimate \( \hat{\mathbf{H}} \) is communicated back to the transmitter via the (ideal) feedback link of Fig.1.

C. The Payload Phase

Thus, on the basis of the available \( K_d \) and \( \hat{\mathbf{H}} \) matrices and actual message \( M \) to be transmitted (see Fig.1), the transmit \( T_x \) node of Fig.1 suitable shapes the (random) signal streams \{\( \phi_i(n) \in \mathbb{C}^1, T_L + T_{tr} + 1 \leq n \leq T \)\}, \( 1 \leq i \leq t \), to be radiated during the payload phase. The corresponding (sampled) signals \{\( y_j(n) \in \mathbb{C}^1, T_L + T_{tr} + 1 \leq n \leq T \)\}, \( 1 \leq j \leq r \), measured at the outputs of the receive antennas may be modelled as

\[ y_j(n) = \frac{1}{\sqrt{t}} \sum_{i=1}^{t} h_{ji} \phi_i(n) + \tilde{d}_j(n), \quad T_L + T_{tr} + 1 \leq n \leq T, \quad 1 \leq j \leq r, \quad (10) \]
where the sequences \( d_j(n) \triangleq v_j(n) + w_j(n), 1 \leq j \leq r \), account for the overall disturbances experienced during the payload phase. They still exhibit the same statistics previously detailed in (4), (5) and they may be assumed independent from both path gains \( \{h_{ji}\} \) and payload streams \( \{\phi_j\} \). Therefore, after assuming that these last meet the (usual) power constraint

\[
\frac{1}{t} \sum_{i=1}^{t} \mathbb{E}\{||\phi_i(n)||^2\} = P, \ T_L + T_{tr} + 1 \leq n \leq T,
\]

the resulting SINR \( \gamma_j \) measured at the output of the \( j \)-th receive antenna during the payload phase equates (see eqs. (5), (10))

\[
\gamma_j = \frac{P}{(N_0 + c_{jj})}, \ 1 \leq j \leq r.
\]

Furthermore, from (10) we also deduce that the \((r \times 1)\) column vector \( y(n) \triangleq [y_1(n) ... y_r(n)]^T \) collecting the outputs of the \( r \) receive antennas over the \( n \)-th payload slot is linked to \((t \times 1)\) column vector \( \phi(n) \triangleq [\phi_1(n) ... \phi_t(n)]^T \) of the corresponding signals radiated by the \( T_x \) node as in

\[
y(n) = \frac{1}{\sqrt{t}} H^T \phi(n) + d(n), \ T_L + T_{tr} + 1 \leq n \leq T,
\]

where \( \{d(n) \triangleq [d_1(n) ... d_r(n)]^T, \ T_L + T_{tr} + 1 \leq n \leq T\} \) is the temporally white Gaussian sequence of disturbances with spatial covariance matrix still given by \( K_d \) in (5). Furthermore, directly from (10.1) it follows that the \((t \times t)\) spatial covariance matrix \( R_{\phi} \triangleq \mathbb{E}\{\phi(n)\phi(n)^\dagger\} \) of the \( t \)-dimensional signal radiated during each slot must meet the power constraint

\[
Tra[R_{\phi}] \equiv \mathbb{E}\{\phi(n)\phi(n)^\dagger\} = tP, \ T_L + T_{tr} + 1 \leq n \leq T.
\]

Finally, after stacking the \( T_{pay} \) observed vectors in (11) into the corresponding \((T_{pay}r \times 1)\) block vector \( \bar{Y} \triangleq [y^T(T_L + T_{tr} + 1) ... y^T(T)]^T \), we may compact the \( T_{pay} \) relationships (11) in the following one:

\[
\bar{Y} = \frac{1}{\sqrt{t}} [I_{T_{pay}} \otimes H]^T \bar{\phi} + \bar{d},
\]

where the (block) covariance matrix of the corresponding disturbance block vector in (12) \( \bar{d} \triangleq [d^T(T_L + T_{tr} + 1) ... d^T(T)]^T, \) equates

\[
\mathbb{E}_j\{\bar{d}^\dagger \bar{d}\} = I_{T_{pay}} \otimes K_d
\]

\( \hat{5} \)We point out that our model explicitly accounts for the different power levels \( \tilde{P} \) and \( P \) possibly radiated by transmit antennas during the training and payload phases, respectively.

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while the block vector $\mathbf{\phi} \triangleq [\mathbf{\phi}^T (T_L + T_{tr} + 1) \ldots \mathbf{\phi}^T (T)]^T$ of the random signals transmitted during overall payload phase is constrained as in (see (11.1))

$$E\{\mathbf{\phi}^T \mathbf{\phi}\} = T_{pay} P.$$ (12.2)

III. MMSE MIMO Channel Estimation in the Presence of Spatially Colored MAI

Since in [9] it is proved that the MMSE estimate $\hat{\mathbf{H}} \equiv [\hat{h}_1 \ldots \hat{h}_r] \triangleq E\{\mathbf{H} | \tilde{\mathbf{Y}}\}$ of the MIMO channel $\mathbf{H}$ in (9) is a sufficient statistic for the ML detection $\mathcal{M}$ of the transmitted message $\mathcal{M}$, no information loss is paid by the here considered receive architecture that it is composed by the MIMO MMSE channel estimator cascaded to the ML detector of the transmitted message $\mathcal{M}$. Thus, passing now to develop the structure of the MMSE MIMO channel estimator, we begin to note that, due to the spatial coloration induced by the MAI, all columns of the observed matrix $\tilde{\mathbf{Y}}$ in (9) are mutually correlated. So, unlike to the well known case of spatially white MAI afforded, for example, in [6,9], in the considered environment each MMSE path gain estimate $\hat{h}_{ji}$ generally depends on the overall matrix $\tilde{\mathbf{Y}}$ of the $(T_{tr} \times r)$ observations received during the training phase. Thus, a suitable application of the Principle of Orthogonality leads to the following expression for the MMSE estimates $\hat{h}_j \triangleq E\{h_j | \tilde{\mathbf{Y}}\}$ of the $j$-th column of matrix $\mathbf{H}$ (see the Appendix A):

$$\hat{h}_j = \frac{1}{\sqrt{t}} \left[ e_j^T K_{d}^{-1/2} \otimes \tilde{\mathbf{X}} \right] \left[ \frac{1}{t} \left( K_{d}^{-1} \otimes \mathbf{X} \mathbf{X}^\dagger + I_{r T_{tr}} \right) \right]^{-1} \left( K_{d}^{-1/2} \otimes I_{T_{tr}} \right) \text{ vect}(\tilde{\mathbf{Y}}),$$

$$1 \leq j \leq r,$$ (13)

where the $r$-dimensional (columnn) vector $e_j$ is the (usual) $j$-th unit vector of $\mathbb{R}^t$ [13], $K_{d}^{-1/2}$ indicates the positive square root of the matrix $K_{d}^{-1}$ [13], while vect$(\tilde{\mathbf{Y}})$ is the $r T_{tr}$-dimensional column vector given by the ordered stacking of the columns of the observed matrix $\tilde{\mathbf{Y}}$ in (9). Furthermore, the cross-correlation matrices of the columns of the resulting MMSE error matrix $\mathbf{E} \equiv [\epsilon_1 \ldots \epsilon_r] \triangleq \mathbf{H} - \hat{\mathbf{H}}$ are given by

$$E \left\{ \epsilon_j (\epsilon_i^\dagger) \right\} = \delta(j, i) I_t - E \left\{ \hat{h}_j (\hat{h}_i^\dagger) \right\} =$$

$$= \delta(j, i) I_t - \frac{1}{t} (e_j \otimes I_t) \left( K_{d}^{-1/2} \otimes \tilde{\mathbf{X}} \right) \left[ \frac{1}{t} \left( K_{d}^{-1} \otimes n \tilde{\mathbf{X}} \mathbf{X}^\dagger + I_{r T_{tr}} \right) \right]^{-1} \cdot \left( K_{d}^{-1/2} \otimes \tilde{\mathbf{X}} \right) (e_i \otimes I_t),$$

$$1 \leq j, i \leq r$$ (14)

so that the resulting total mean square error $\sum_{tot} \triangleq ||\mathbf{E}||_F^2$ equates (see (14) for $j=i$)

$$\sum_{tot} \triangleq \sum_{j=1}^{r} T_{ra} \left[ \epsilon_j \epsilon_j^\dagger \right] = rt +$$
\[-\frac{1}{l} \sum_{j=1}^{r} \text{Tr} \left[ (\epsilon_j \otimes I_l)^\dagger \left( \left( K_d^{-1/2} \otimes \tilde{X} \right)^\dagger \left( \frac{1}{l} \left( K_d^{-1} \otimes XX^\dagger \right) + I_r Tr_r \right)^{-1} \cdot \left( K_d^{-1/2} \otimes \tilde{X} \right) \left( \epsilon_j \otimes I_l \right) \right) \right]. \tag{15} \]

A. The Condition for the Optimal Training

Since the total mean square error \( \sum_{\text{tot}} \) in (15) depends on the matrix \( \tilde{X} \) employed for the training, the key-problem becomes how to choose \( \tilde{X} \) to minimize (15) under the power constraint (9.1). By fact, an application of Cauchy inequality [13] leads to the following condition characterizing the optimal \( \tilde{X} \) (see the Appendix B).

**Proposition 1.** A training matrix \( \tilde{X} \) minimizes the total squared error (15) under the power constraint (9.1) if and only if it meets the following relationship:

\[ K_d^{-1} \otimes \tilde{X}^\dagger \tilde{X} = a I_r, \tag{16} \]

where the positive scalar \( a \) equates

\[ a \triangleq \frac{T_{tr} \hat{P}}{r} \text{Tr} [K_d^{-1}]. \tag{16.1} \]

The relationship (16) points out that the optimized training matrix \( \tilde{X} \) depends on the spatial coloration exhibited by the MAI via the corresponding covariance matrix \( K_d \). When this last equates the identity matrix (e.g., when the MAI is spatially white), eq.(16) becomes \( \tilde{X}^\dagger \tilde{X} = a I_r \), so that, in this case, the optimized \( \tilde{X} \) becomes (para) unitary [9,10].

B. The MMSE MIMO Channel Estimator for the Optimal Training

By introducing (16) in (13) and (14), these last simplify as in

\[ \hat{h}_j = \frac{1 - \sigma^2}{\sqrt{t}} \left( e_j^T K_d^{-1} \otimes \tilde{X} \right) \text{vect}(\tilde{Y}), \quad 1 \leq j \leq r, \tag{17} \]

\[ E\{e_j | e_i|^\dagger\} = \delta(j, i) I_r - E\{\hat{h}_j | h_i\}^\dagger \equiv \sigma^2 e I_r \delta(j, i), \quad 1 \leq j, i \leq r, \tag{18} \]

where

\[ \sigma^2 \triangleq E\{||e_j||^2\} \equiv E||h_{ji} - \hat{h}_{ji}||^2 = \left( 1 + \frac{a}{t} \right)^{-1}, \quad 1 \leq j, i \leq r, \tag{19} \]

indicates the common value of the mean square error affecting the MMSE estimate \( \hat{h}_{ji} \) of each path gain \( h_{ji} \). The relationships (17), (18), (19) point out that the utilization of the optimized training matrix \( \tilde{X} \) strongly simplifies the structure of the corresponding MMSE MIMO channel estimator\(^6\). In addition, it

\(^6\)The row vector \( e_j^T K_d^{-1} \) in (17) equates the j-th row of \( K_d^{-1} \).
makes the \((r \times t)\) Gaussian scalar components \(\{\hat{h}_{ji}\}\) of the estimated channel matrix \(\hat{H}\) uncorrelated and equi-distributed, so that the corresponding pdf \(p(\hat{H})\) assumes the following simple form:

\[
p(\hat{H}) = \left(\frac{1}{\pi(1 - \sigma^2_\epsilon)}\right)^{rt} \exp \left\{ - \frac{1}{(1 - \sigma^2_\epsilon)} \text{Tr}[\hat{H}^\dagger \hat{H}] \right\}. \tag{20}
\]

Finally, also the \((r \times t)\) Gaussian elements \(\{\epsilon_{ji}\}\) of the resulting MMSE error matrix \(E\) are uncorrelated and equi-distributed and their variances equate \((19)\). Furthermore, directly from \((17)\), \((19)\) we also conclude that the channel estimate \(\hat{H}\) approaches actual value \(H\) of the MIMO channel for \(\sigma^2_\epsilon \to 0\), while \(\hat{H}\) vanishes for \(\sigma^2_\epsilon \to 1\) so that the following limits hold

\[
\lim_{\sigma^2_\epsilon \to 0} \hat{H} = H \quad \text{\((21.1)\)}; \quad \lim_{\sigma^2_\epsilon \to 1} \hat{H} = 0_{t \times r}. \quad \text{\((21.2)\)}
\]

According to a current taxonomy, we qualify as Perfect CSI (PCSI) and NoCSI (NCSI) the above limit operating conditions \((21.1), (21.2)\) respectively, while Imperfect CSI (ICSI) refers to the middle case of \(0 < \sigma^2_\epsilon < 1\). Since, after the learning phase, the covariance matrix \(K_d\) is available at both transmit and receiving nodes, the foregoing throughput analysis and power allocation will be carried out by assuming that the training matrix \(\tilde{X}\) meets the optimality condition in \((16)\).

IV. CONVEYED INFORMATION THROUGHPUT IN THE PRESENCE OF CHANNEL ESTIMATION ERRORS AND SPATIALLY COLORED MAI

The block-fading model introduced in Sect.II for the forward MIMO channel of Fig.1 guarantees that this last is information stable \([16]\), so that the corresponding Shannon Capacity \(C\) fixes the maximum information throughput conveyable in a reliable way during the payload phase. Following quite standard approaches \([14]\), the capacity \(C\) of the MIMO channel \((12)\) can be expressed as

\[
C = \{C(\hat{H})\} \equiv \int \{C(\hat{H})\} p(\hat{H}) d\hat{H}, \quad \text{(nats/payload slot),} \tag{21}
\]

where \(p(\hat{H})\) is given by \((20)\) and the random variable

\[
C(\hat{H}) \triangleq \sup_{\varphi \in \tilde{X}} E \left( \frac{1}{T_{\text{pay}}^T} I \left( \tilde{y}_r; \tilde{\varphi}_r \left| \hat{H} \right. \right) \right), \tag{22}
\]

is the capacity of the MIMO channel \((12), (12.2)\) conditioned on the realization \(\hat{H}\) of the channel estimate actually available at both transmitter and receiver of Fig.1. Finally, \(I(\cdot;\cdot;\cdot)\) in \((22)\) is the mutual information conveyed by MIMO channel \((12)\) during the payload phase. Unfortunately, barring two limit cases of PCSI \([1,2,4,5]\) and NCSI \([6,7]\), the pdf of the input signals \(\tilde{\varphi}\) achieving the supremum in \((22)\) is currently unknown even for the simplest case of spatially white MAI. However, it is...
known that Gaussian distributed input signals achieve the supremum in (22) not only when the condition (21.1) of PCSI is approached [1,2,4,5], but also for $0 < \sigma_\epsilon^2 < 1$ when the length $T_{\text{pay}}$ of the payload phase (largely) exceeds the number $t$ of transmit antennas (see [7] about this asymptotic important result). Therefore, motivated by the above considerations, in the following we focus on the evaluation of (22) for Gaussian distributed input signals. In this case the $T_{\text{pay}}$ components $\{\phi(n) \in \mathbb{C}^t, T_L + T_{tr} + 1 \leq n \leq T\}$ in (11) of the overall signal vector $\overrightarrow{\phi}$ in (12) are modeled as uncorrelated zero-mean proper complex Gaussian vectors with correlation matrix $R_\phi$ meeting (11.1). Obviously, the corresponding information throughput

$$T_G(\hat{H}) \triangleq \frac{1}{T_{\text{pay}}} \sup_{|R_\phi| \leq P_\epsilon} I\left(\overrightarrow{\bar{y}}; \overrightarrow{\phi} | \hat{H}\right), \quad (23)$$

coupled by the MIMO channel (12) generally falls below $C(\hat{H})$ in (22), so that we have $T_G(\hat{H}) \leq C(\hat{H})$. However, the above inequality is satisfied as equality when at least one of the above cited operating conditions is met. Therefore, passing now to deal with the evaluation of $T_G(\hat{H})$ in (23), we remark that, in general, the conditional mutual information $I\left(\overrightarrow{\bar{y}}; \overrightarrow{\phi} | \hat{H}\right)$ in (23) resists closed-form computation. However, the considerations and analytical developments detailed in the Appendix C lead to the following closed-form result.

**Proposition 2.** Let us assume the training matrix $\tilde{X}$ meeting the relationship (16) and also assigned the spatial correlation matrix $R_\phi$ in (11.1) of the payload streams radiated by transmit antennas. Thus, the resulting conditional mutual information $I\left(\overrightarrow{\bar{y}}; \overrightarrow{\phi} | \hat{H}\right)$ in (23) supported by the MIMO channel (12) admits the following closed-form expression:

$$I\left(\overrightarrow{\bar{y}}; \overrightarrow{\phi} | \hat{H}\right) = T_{\text{pay}} \log \det \left[ \left( I_r + \frac{1}{t} K_d^{-1/2} \hat{H}^T R_\phi \hat{H} K_d^{-1/2} + \sigma_\epsilon^2 P K_d^{-1} \right) \right]$$

$$- \log \det \left[ \left( I_r + \frac{\sigma^2 T_{\text{pay}}}{t} (K_d^{-1})^* \otimes R_\phi \right) \right] \quad (24)$$

when at least one of the conditions listed below is met:

a) both $T_{\text{pay}}$ and $t$ are large; \hspace{1cm} (24.1)

b) $\sigma_\epsilon^2$ vanishes; \hspace{1cm} (24.2)

c) all SINRs $\gamma_j$, $1 \leq j \leq r$, in (10.2) vanish. \hspace{1cm} \diamond (24.3)

The analytical developments reported in the Appendix C show that (24) arises from a combined exploitation of Law of the Large Numbers [26] and Central Limit Theorem and it reduces to eq.(1) of [2] for vanishing $\sigma_\epsilon^2$. Furthermore, we have experienced that the condition (24.1) may be considered, by fact, met for $T_{\text{pay}} \geq 6t \div 7t$ and $t \geq 4 \div 5$ even for $\sigma_\epsilon^2$ approaching unit and SINRs exceeding 6dB-7dB.
A. Optimized Power allocation in the presence of colored MAI and Channel Estimation errors

Therefore, according to (23), we must proceed to the power constrained maximization of the conditional throughput (24). For this purpose, let us indicate as

$$K_d = U_d \Lambda_d U_d^\dagger,$$

the Singular Value Decomposition (SVD) of the MAI spatial covariance matrix $K_d$, where

$$\Lambda_d \triangleq diag\{\mu_1, ..., \mu_r\},$$

is the corresponding $(r \times r)$ diagonal matrix of the magnitude-ordered singular values. Thus, after introducing the $(t \times r)$ matrix

$$A \triangleq \hat{H}^* K_d^{-1/2} U_d,$$

accounting for the combined effects of the imperfect channel estimate $\hat{H}$ and MAI spatial coloration $K_d$, let us denote as

$$A = U_A D_A V_A^\dagger,$$

the related SVD, where $U_A$ and $V_A$ are unitary matrices, while

$$D_A \triangleq diag\{k_1, ..., k_s, 0_{t-s}\},$$

is the corresponding $(t \times r)$ diagonal matrix collecting the $s \triangleq \min\{r, t\}$ magnitude-ordered singular-values $k_1 \geq k_2 \geq ... \geq k_s > 0$ of the matrix $A$. Finally, for future convenience, let us also introduce the following dummy positions:

$$\alpha_m \triangleq \frac{\mu_m k_m^2}{t(\mu_m + P\sigma_z^2)}, \quad 1 \leq m \leq s; \quad \beta_l \triangleq \frac{\sigma_z^2 T_{pay}}{t\mu_l}, \quad 1 \leq l \leq r.$$

Thus, the application of the Kuhn-Tucker conditions \[14, eqs.(4.4.10), (4.4.11)\] allows us to evaluate the optimized transmit powers $\{P^*(m), 1 \leq m \leq t\}$ achieving the constrained sup in (23) as detailed in the following Proposition 3 (see the Appendix D).

**Proposition 3.** Let us assume that at least one of the above operating conditions (24.1), (24.2), (24.3) is fulfilled. Thus, for $m = s + 1, ..., t$, powers achieving the sup in (23) vanish, while for $m = 1, ..., s$ they must be computed according to the following two relationships:

$$P^*(m) = 0, \text{ when } k_m^2 \leq \left(1 + \frac{\sigma_z^2 P}{\mu_m}\right)\left(\frac{l}{\rho} + \frac{\sigma_z^2 Tr[K_d^{-1}]}{\mu_l}\right),$$

$$P^*(m) = \frac{1}{2\beta_{min}} \left\{\beta_{min} \left[\left(1 - \frac{r}{T_{pay}}\right)\rho - \frac{1}{\alpha_m}\right] - 1+ \right\}.$$
\[
+ \sqrt{\left\{ \beta_{\text{min}} \left[ \left( 1 - \frac{r}{T_{\text{pay}}} \right) \rho - \frac{1}{\alpha_m} \right] - 1 \right\}^2 + 4 \beta_{\text{min}} \left( \rho - \frac{1}{\alpha_m} \frac{r \rho \beta_{\text{min}}}{T_{\text{pay}}} \right)},
\]
when \(k_m^2 > \left( 1 + \frac{\sigma^2_p \rho}{\mu_m} \left( \frac{t}{\rho} + \sigma^2_e \text{Tr} \left[ K^{-1} \right] \right) \right)^2 + 4 \beta_{\text{min}} \left( \rho - \frac{1}{\alpha_m} \frac{r \rho \beta_{\text{min}}}{T_{\text{pay}}} \right),\) (29)

where \(\beta_{\text{min}} \triangleq \min \{ \beta_l, l = 1, \ldots, r \}.\) Furthermore, the nonnegative scalar parameter \(\rho\) in (28), (29) is set so to satisfy the power constraint (see eq.(11.1))

\[
\sum_{m \in \mathcal{I}(\rho)} P^*(m) = P_t,
\]
(30)

where

\[
\mathcal{I}(\rho) \triangleq \{ m = 1, \ldots, s : k_m^2 > \left( 1 + \frac{\sigma^2_p \rho}{\mu_m} \left( \frac{t}{\rho} + \sigma^2_e \text{Tr} \left[ K^{-1} \right] \right) \right) \},
\]
(30.1)
is the (\(\rho\)-depending) set of indexes fulfilling the inequality (29). Finally, the corresponding optimized spatial correlation matrix for the radiated signals is aligned along the right-eigenvectors of the matrix \(A\) in (26.1) according to

\[
\mathbf{R}_\phi(\text{opt}) = \mathbf{U}_A \text{diag} \{ P^*(1), \ldots P^*(s), 0 \}_{s \leftarrow s} \mathbf{U}_A^\dagger,
\]
(31)

so that the resulting maximized throughput in (23) admits the following (simple) closed-form expression:

\[
\mathbb{T}_{G}(\hat{\mathbf{H}}) = \sum_{m=1}^{r} \log \left( 1 + \frac{\sigma^2_e P}{\mu_m} \right) + \sum_{m=1}^{s} \left[ \log(1 + \alpha_m P^*(m)) - \frac{1}{T_{\text{pay}}} \sum_{l=1}^{r} \log \left( 1 + \beta_l P^*(m) \right) \right].
\]
(32)

About the above reported power allocation, some remarks are in order. First, the considerations reported in the last part of the Appendix D support the conclusion that the power allocation (28),(29) is still optimal in a min-max sense also when all above stated operating conditions fall short. Second, an exploitation of the (truncated) expansion: \(\sqrt{1 + x} \approx 1 + 0.5x\) allows us to rewrite (28), (29) in the following form for vanishing \(\sigma^2_e\):

\[
\lim_{\sigma^2_e \to 0} P^*(m) = \max \left\{ 0, \rho - \frac{t}{k_m^2} \right\}, m = 1, \ldots, s,
\]
(33)

and this last agrees with the water-filling power allocation previously reported in [1,2] for the case of perfect channel estimation. Third, when \(\sigma^2_e\) approaches unit, then the channel estimate \(\hat{\mathbf{H}}\) vanishes (see(21.2)), so that no information is available at both transmitter and receiver about actual values assumed by the MIMO channel path gains \(\{ h_{ji} \}.\) So, after evaluating (24) for \(\sigma^2_e \to 1\), in this limit case we arrive at the following limit expression for the sustained throughput \(\mathbb{T}_{G}(\hat{\mathbf{H}})\) in (23):

\[
\lim_{\sigma^2_e \to 1} \mathbb{T}_{G}(\hat{\mathbf{H}}) \triangleq \mathbb{T}_{G}(\mathbf{0}) = \sum_{m=1}^{r} \log \left( \frac{1 + P}{\mu_m} \right) \left( \frac{1 + P_{T_{\text{pay}}} \mu_m}{1 + P_{T_{\text{pay}}} \mu_m} \right)^{1/T_{\text{pay}}} \right], \text{ (nats/payload slot)}.
\]
(34)

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Since this last holds for large \( t \) and \( T_{\text{pay}} \) regardless of radiated power \( P \), the relationship (34) directly supports the conjecture originally reported in [7] about the capacity-achieving property retained for large \( T_{\text{pay}} \) by the input Gaussian pdf even in application scenarios with fully non-coherent detection. Interesting enough, this limit property exhibited by (34) is no retained by some recently published lower bounds on \( T_G(\hat{H}) \) derived according to the principle of the so-called “Synchronized Detection” [10 and reference therein]. To shortly summarize above considerations about the validity limits of the relationship (32), we may conclude that for vanishing \( \sigma^2_\varepsilon \) and/or small SINRs the throughput \( T_G(\hat{H}) \) in (32) approaches actual capacity \( C(\hat{H}) \) in (22) regardless of values assumed by \( T_{\text{pay}} \) and \( t \). Furthermore, we have ascertained via numerical trials that, for \( 0 < \sigma^2_\varepsilon \leq 1 \), \( T_G(\hat{H}) \) in (32) is indistinguishable from actual \( C(\hat{H}) \) in (22) at medium/large SINRs when the number \( t \) of transmit antennas exceeds 4-5 units and the payload length \( T_{\text{pay}} \) is over 6\( t \).

B. A Numerical Algorithm for implementing the Optimized Power Allocation

Passing now to shortly consider the numerical implementation of the presented power allocation formulas, the first step for computing (28), (29) relies on the evaluation of \( \rho \) satisfying the relationship (30). Although this computation resists closed-form analytical computation also in the simplest case of PCSI [1,2], nevertheless we observe that the cardinality \( |J(\rho)| \) of the indexes set in (30.1) vanishes at \( \rho = 0 \) and then it increases for growing \( \rho \). This means that the solution of (30) may be found by implementing a (very) simple iterative procedure that starts with \( \rho = 0 \) and then progressively increases the current value of \( \rho \) by a pre-assigned step-size\(^7\) \( \Delta \) until summation in (30) equates the power constraint \( P_t \). The resulting algorithm for the numerical evaluation of the optimized powers (28), (29) is summarized in the Table I and several performance plots obtained via its computer-implementation will be presented in Sect.VI.

V. A Topology-Based MAI model for Multi-Antenna Ad-hoc Networks

To test the effectiveness of the proposed power allocation algorithm, we consider the application scenario sketched in Fig.3 that may be considered adequate to capture the key-features of the spatial MAI impairing emerging Multi-Antenna ad-hoc networks [15,18,20]. In particular, we assume that the network of Fig.3 is composed by \((N+1)\) no cooperative, mutually interfering, point-to-point links \( T_x f \rightarrow R_x f, 0 \leq f \leq N \), so that the overall signal received by the reference node \( R_x 0 \) is the superposition

\(^7\)Several carried out numerical trials confirmed that \( \Delta = 0.1 P_t \) suffices to evaluate \( \rho \) with adequate accuracy.
of the desired signal generated by $T_x0$ with the $N$ interfering signals radiated by remaining transmit
nodes $T_xf, 1 \leq f \leq N$. Each transmit node is equipped with $t_f, 0 \leq f \leq N$, transmit antennas while
$r_f, 0 \leq f \leq N$, is the number of receive antennas located at the corresponding $R_x0$ node. Thus, after
indicating as $l_f$ the distances $T_x0 \rightarrow R_x0, 0 \leq f \leq N$, the $r_0$-dimensional disturbance $d(n)$ in (11)
received by $R_x0$ during the $n$-th slot of the payload phase may adequately modelled as
\begin{equation}
\begin{align*}
d(n) &= \sum_{f=1}^{N} \sqrt{\left(\frac{l_0}{l_f}\right)^4} \frac{1}{\sqrt{l_f}} \chi_f H_fT \phi^{(f)}(n) + w(n),
\end{align*}
\end{equation}
where $w(n)$ accounts for the thermal noise present in $R_x0$ (see(11)) $\phi^{(f)}(n)$ is the $t_f$-dimensional
(Gaussian) signal radiated by $f$-th interfering node $T_xf$, $\chi_f$ accounts for the shadowing effects and the
$(t_f \times r_0)$ matrix $H_f$ describes the Ricean-distributed fast-fading phenomena affecting $f$-th interfering link
$T_xf \rightarrow R_x0$. More in detail, according to the fast fading spatial interference model recently developed
in [1,2], $f$-th matrix channel response $H_f$ in (35) may be expressed as
\begin{equation}
\begin{align*}
H_f &= \left[ \frac{k_f}{1 + k_f} H^{(sp)}_f + \frac{1}{1 + k_f} H^{(sc)}_f \right], 1 \leq f \leq N,
\end{align*}
\end{equation}
where $k_f \in [0, +\infty)$ is the $f$-th Ricean factor and the elements of the $(t_f \times r_0)$ matrix $H^{(sc)}_f$ are mutually
independent zero-mean unit-variance proper Gaussian r.v.s accounting for the scattering component of
the $f$-th interfering link $T_xf \rightarrow R_x0$ of Fig.3. In turn, the $(t_f \times r_0)$ specular component matrix $H^{(sp)}_f$
of $f$-th interfering link may be computed as \[1,2]\n\begin{equation}
\begin{align*}
H^{(sp)}_f = a(f) b(f)^T, 1 \leq f \leq N,
\end{align*}
\end{equation}
where $a(f)$ and $b(f)$ are the $(r_0 \times 1)$ and $(t_f \times 1)$ column vectors describing the specular array responses
at the receiving node $R_x0$ and transmit one $T_xf$ for the $f$-th interfering link, respectively \[1,2\]. When
regularly spaced linear arrays with isotropic elements are employed both at $T_xf$ and $R_x0$, above vectors
may directly evaluated as in \[1,2,15\]
\begin{equation}
\begin{align*}
a(f) &= [1, \exp(j2\pi\nu \cos(\theta^{(f)}_a)), ... \exp(j2\pi\nu(r_0 - 1) \cos(\theta^{(f)}_a))]^T,
\end{align*}
\end{equation}
\[8\]To simplify the adopted notation, we assume that all $N$ interfering links $T_xf \rightarrow R_xf, 1 \leq f \leq N$, in Fig.3 are in the
payload phase. Generalization of (35) to the most general case where only $M$ out of $N$ interfering links radiate payload signals
is direct and only requires minor formal modifications of (35).
\[9\]Without loss of generality we may assume the r.v. $\chi_f$ spanning the interval $[0, 1]$. This means that $\chi_f = 1$ corresponds to
the worst case when the impairing effects of MAI generated by the transmit node $T_xf$ are maximal while $\chi_f = 0$ describes the
lucky operating condition where no interference is experienced by the receive node $R_x0$ of Fig.3 due to the signals radiated by
$T_xf$. 

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\[ \mathbf{b}(f) = [1, \exp(j2\pi \nu \cos(\theta_d^{(f)})), \ldots, \exp(j2\pi \nu (t_f - 1) \cos(\theta_d^{(f)}))]^T, \]  

(36.3)

where \( \theta_a^{(f)}, \theta_d^{(f)} \) are the arrival and departure angles for the \( f \)-th interfering link (see Fig.3), while \( \nu \) is the antenna spacing in multiple of RF radiated wavelengths\(^{10}\).

A. Model for the MAI Covariance Matrix

Therefore, after assuming that the spatial covariance matrix \( \mathbf{R}^{(f)}_{\phi} \triangleq \mathbb{E}\{\phi^{(f)}(n)\phi^{(f)}(n)^\dagger\}, 1 \leq f \leq N \), of the signals radiated by \( f \)-th transmit node \( T_x f \) is power-constrained as in (see(11.1))

\[ \text{Tr} \{ \mathbf{R}^{(f)}_{\phi} \} = t_f P^{(f)}, \]  

(37)

the covariance matrix \( \mathbf{K}_d \) for the MAI model described by (35) equates

\[ \mathbf{K}_d \triangleq \mathbb{E}\{\mathbf{d}(n)\mathbf{d}(n)^\dagger\} = \left\{ \begin{array}{c} N_0 + \sum_{f=1}^{N} \left( \frac{k}{l_f} \right)^4 \mathbb{E}\{x^2\} \{P^{(f)}\} \mathbf{I}_r + \left\{ \sum_{f=1}^{N} \left( \frac{k}{l_f} \right)^4 \mathbb{E}\{x^2\} \{a(f)\mathbf{b}^T(f)\mathbf{R}^{(f)}_{\phi} \mathbf{b}^*(f)a^\dagger(f)\} \right\} \end{array} \right\} \]  

(38)

This relationship represents the main results of this Section because it accounts for the MAI effects arising from main topological and propagation features of typical Multi-Antenna ad-hoc networks. In particular, eq.(38) stresses that MAI interference may be considered spatially white when all the Ricean factors of the interfering links vanish. On the contrary, for moderate-to-large Ricean factors the spatial coloration exhibited by the MAI may be substantial, as confirmed by the numerical results of the next Section.

B. A Worst-Case Application Example

As an application example, let us consider the hexagonal ad-hoc network sketched in Fig.4, where all transmit and receive nodes are equipped with the same number of transmit and receive antennas (e.g., \( t_0 = t_1 = t_2 = t \) and \( r_0 = r_1 = r_2 = r \)) and all transmit nodes radiate the same power level (e.g., \( P_0 = P_1 = P_2 = P \)). After assuming the array elements spaced apart by one-half wavelength (e.g., \( \nu=1/2 \)) and all Ricean factors sharing a same value (e.g., \( k_1 = k_2 = k \)), let us consider a worst-operating condition where all the shadowing coefficients are unit (e.g., \( \chi_1 = \chi_2 = 1 \)) and, in addition, both correlation matrices \( \mathbf{R}^{(1)}_{\phi}, \mathbf{R}^{(2)}_{\phi} \) of the spatial signals radiated by the interfering transmit nodes \( T_x 1, T_x 2 \) equate \( PI_t \{ 1, 2 \} \). In this case eq.(38) becomes

\[ \mathbf{K}_d = \{ N_0 + \frac{2}{9} \frac{P}{1 + k} \mathbf{I}_r + \left\{ \frac{k}{1 + k} \frac{P}{9} \sum_{f=1}^{2} a(f)\mathbf{b}^T(f)\mathbf{b}^*(f)a^\dagger(f) \right\} \}, \]  

(39)

\(^{10}\)Several measures support the conclusion that \( \nu \) values of the order of 1/2 generally suffice to meet above mentioned uncorrelation assumption between rays impinging receive antennas in application scenarios as those here considered with terminals (approximately) co-located at the same level over the ground [15].
where

\[ a(1) = [1, \exp(j\pi \frac{\sqrt{3}}{2}), ..., \exp(j\pi (r-1) \frac{\sqrt{3}}{2})]^T, \]

\[ b(2) = [1, \exp(j\pi \sqrt{3}), ..., \exp(j\pi (t-1) \frac{\sqrt{3}}{2})]^T, \]

and \( b(1), a(2) \) are column vectors composed by \( t \) and \( r \) unit elements, respectively.

VI. Numerical Results and Performance Comparisons

Although the joined pdf of the \((rxt)\) elements of the channel estimate \( \hat{H} \) assumes the (simple) expression in (20), nevertheless the corresponding expectation

\[ T_G \triangleq E\{T_G(\hat{H})\}, \]

of the conditional throughput in (32) resists closed-form analytical evaluation even in the simplest case of spatially white MAI with vanishing \( \sigma_e^2 \) [4,5,17 and reference therein]. Thus, as in [1,2,4] in this Section we resorts to a Monte-Carlo approach for computing the expectation (40) based on the sample-average of 10,000 independent realizations of the conditional throughput \( C_G(\hat{H}) \). Furthermore, all the reported numerical plots refer to the hexagonal network of Sect.V.B with \( N_0 \) in (39) set to unit and various values for the power level \( P \) radiated by transmit nodes.

A. Effects of the channel estimation errors

Goals of the first set of plots drawn in Fig.5 is to test the sensitivity of the average throughput \( C_G \) of the reference link \( T_{x0} \rightarrow R_{x0} \) of Fig.4 on the channel estimation errors affecting the corresponding available MIMO channel estimate \( \hat{H} \). All nodes of the simulated system of Fig.4 are assumed equipped with \( r=t=8 \) antennas, all the Ricean factor \( k \) in (39) are set to 10 and length \( T_{pay} \) of the payload phase equal to 40 is considered. An examination of Fig.5 leads to the conclusion that the channel estimation error variance \( \sigma_e^2 \) does not induce considerable performance loss till values around 0.01.

B. Effects of the number of transmit/receive antennas

The numerical plots drawn in Fig.6 allow us to evaluate the effect on the MAI of the number \( r=t \) of antennas equipping each node of the network of Fig.4. More in detail, these plots report the average throughput (40) of the reference link \( T_{x0} \rightarrow R_{x0} \) of Fig.4 when the Ricean factor \( k \) in (39) equates 10, and \( \sigma_e^2 = 0.01, T_{pay} = 80 \). An examination of plots of the Fig.6 leads to the conclusion that by increasing the number of antennas we are able to quickly gain in terms of channel capacity, and the resulting increment becomes greater at higher values of \( t \) and \( r \).
C. Effects of mismatch in the evaluation of the MAI Covariance Matrix

As anticipated in Sect.II.A, accuracy in the estimate $K_d$ in (6) of actual MAI covariance matrix $K_d$ is mainly limited by the length $T_L$ of the employed learning phase. To test the sensitivity of proposed power allocation algorithm of Table I on errors possibly affecting the estimated matrix $\hat{K}_d$, we have perturbed actual $K_d$ via a randomly generated $(r \times r)$ matrix $N$ composed by zero-mean proper complex unit-variance independent Gaussian elements. This was accomplished according to the following relationship:

$$\hat{K}_d = K_d + \sqrt{\frac{||K_d||_E^2}{r^2}} \sqrt{\delta} N,$$

where $\delta \triangleq \mathbb{E}[[K_d - \hat{K}_d]/||K_d||_E]^{2}$ in (41) is a deterministic parameter set according to the desired squared estimation errors. Thus, using this perturbed covariance matrix $\hat{K}_d$ in place of actual one $K_d$, we have performed the power allocation as indicated by the relationships (28), (29). Then, we have evaluated the resulting conditional capacity $T_G(\hat{H})$ via eq.(32), with parameters $\{\alpha_m\}$ and $\{\beta_l\}$ computed as in (27) on the basis of actual MAI matrix $K_d$. The obtained average throughputs are plotted in Fig.7 for the reference link $T_x0 \rightarrow R_x0$ of Fig.4 with $T_{pay} = 40$, $r=t=8$, $\sigma^2 = 0.015$, $k = 10$, and various values of average relative gap $\delta$ in (41). The plots of Fig.7 support the conclusion that the throughput loss induced by errors possibly present in the estimated MAI covariance matrix $\hat{K}_d$ are negligible for average relative squared error levels $\delta$ as high as 0.01. In turn, these considerations confirm that, by fact, the adopted assumption of noiseless ideal feedback link is reasonable and not too critical.

D. Coordinated versus Uncoordinated Medium Access strategies

Although in these last years MAI-mitigation capability offered by Multi-Antenna systems based on smart-like technology has been often claimed [8,15,18], nevertheless a still open question concerns the comparison of the information throughputs $T_G$ in (40) conveyed by here considered MAI-impaired systems with those guaranteed by orthogonal MAI-free TDMA (or FDMA)-based access techniques. By fact, till now no firm evidence of superiority of an access technique over the other one is available in the literature, specially for the application scenarios here considered (see Fig.3) where typical SINRs values are of the order of few dBs so that multiuser detection strategies based on iterative subtractive cancellations of MAI tend to fall short [22]. To gain some (preliminary) insight about this important question, we have numerically evaluated the average information throughout $C_{TDMA} \triangleq \mathbb{E}\{C_{TDMA}(H)\}$ (nats/ payload slot) conveyed by the reference link $T_x0 \rightarrow R_x0$ of Fig.4 when the MAI-free TDMA-
based access is implemented\textsuperscript{11}. The obtained numerical plots of Fig. 8 refer to the network of Fig. 4 with $T_{\text{pay}} = 80$, $\sigma_e^2 = 0.1$, $k_e = 1000$ and values of number of transmit/receive antennas ranging from 4 to 12. Although $T_C$ has been evaluated in the worst MAI condition (see (39) and related comments), nevertheless the plots of Fig. 8 show that $T_G$ outperforms the corresponding $C_{TDMA}$, specially at low radiated power levels $P$ and for transceivers equipped with a large number $r=t$ of transmit/receive antennas. The same conclusions may be drawn by an examination of Fig. 9 for the case of Ricean-factor $k$ equal to 0. Thus, we may argue that when $r$ and $t$ increase the proposed spatial-shaping algorithm of Table I allows us to achieve channel capacity higher than one attained by conventional orthogonal access methods.

\textbf{E. MAC Implications}

The underlying conclusion seems to be that allowing multiple users to collide in the time-frequency plan leads to a more effective utilization of the spatial dimension of the receivers that, by fact, over-compensates the throughput loss induced by the collisions occurred in the time-frequency domain. Based on this conclusion, a more effective MAC strategy may be to allow for some forms of hybrid multi-access scheme where (a moderate level of) collisions are tolerable in the time-frequency domain so to fully exploit the space-division access capability exhibited by the Multi-Antenna system. An example of non-orthogonal access technique may be OFDMA with tone-sharing among multiple users, where the degree of tone-sharing increases with the number of transmit/receive antennas. This conclusion demands for new MAC paradigm and related MAC design criteria where the spatial MAC capability of the Multi-Antenna system is carefully exploited.

\textbf{VII. Conclusions}

Main contributions of this paper focused on the optimized training, MIMO channel estimation and related optimized spatial signal-shaping for Multi-Antenna systems impaired by spatially colored MAI.

\textsuperscript{11}According to [22, Sect.VI.C], for any assigned channel estimation matrix $\hat{H}$ the corresponding conditional information throughput $C_{TDMA}(\hat{H})$ has been evaluated by running the algorithm of Table I under the following operating conditions:

i) all shadowing factors in (39) have been zeroed;

ii) the power level $P$ in (39) radiated by $T_x0$ node of Fig. 4 has been replaced by $3P$;

iii) the resulting throughput $T_C(\hat{H})$ in (32) has been reduced by $1/3$.

While condition i) accounts for the (obvious) fact that the TDMA technique is MAI free, the condition iii) reflects the circumstance that in the TDMA operating mode the reference link $T_x0 \rightarrow R_x0$ of Fig. 4 is active over $1/3$ of the overall time. For the same reason, the power level radiated by $T_x0$ over each TDMA slot may be 3 times bigger than that able to be radiated when $T_x0$ continuously accesses to the channel.
From the carried out analysis and presented numerical results two general conclusions outstand.

First, throughput loss induced by (possible) errors in the estimation of the MAI covariance matrix is typically no substantial and then, in this respect, the introduced assumption of ideal feedback link may be considered reasonable. On the contrary, at medium/high SINRs the sensitivity of the conveyed throughput on the estimation errors possibly impairing the MIMO channel-estimate may be no longer negligible, so that the above assumption of ideal feedback link becomes more questionable. However, some results recently presented in [24] leads to the conclusion that, in this case, the throughput analysis carried out in Sect.IV still hold, after replacing actual estimate $\hat{H}$ available at the receiver with the corresponding ones $\hat{\hat{H}}$ computed at the output of the noisy feedback channel (see Fig.1).

Second main conclusion arises from the throughput comparisons of Sect.VI.5. These last confirm the MAI-suppressing capability of Multi-Antenna transceivers even in operating scenarios working in ad-hoc type mode. In particular, the performance plots of Figs.8,9 support for the superiority of uncoordinated spatial-based multiple access techniques over coordinated orthogonal ones relying on more traditional TDMA/FDMA centralized approaches. If this conclusion will be definitely confirmed, it will be of paramount importance for actual implementation and pervasive planning of ad-hoc type high-throughput decentralized networking architectures.

APPENDIX A - THE MMSE MIMO CHANNEL ESTIMATOR

By exploiting the following property of the Kronecker product [13]: $\text{vect}(AB) = [I \otimes A] \text{vect}(B)$, we may rewrite (9) as in

$$\text{vect}(\tilde{Y}) = \frac{1}{\sqrt{t}} [I_r \otimes \tilde{X}] \text{vect}(H) + \text{vect}(\tilde{D}).$$

(A.1)

Therefore, after observing that $E\{\text{vect}(\tilde{D})(\text{vect}(\tilde{D}))^\dagger\} = K_d \otimes I_{T_r}$, and $E\{\text{vect}(\tilde{Y})(\text{vect}(\tilde{Y}))^\dagger\} = \frac{1}{t}(I_r \otimes \tilde{X}\tilde{X}^\dagger) + (K_d \otimes I_{T_r})$, an application of the Orthogonality Principle leads (via somewhat tedious algebra) to eqs. (13), (14). ☐

APPENDIX B - OPTIMIZATION OF THE TRAINING MATRIX
After noting that any training matrix $\tilde{X}$ meeting (16) also satisfies the power-constraint (9.1), an exploitation of the general property \[13\] $\text{Tr}a[A \otimes B] = \text{Tr}a[A] \text{Tr}a[B]$ allows us to rewrite (15) as in
\[
\sum_{\text{tot}} = rt - T_{tr} \tilde{P} \left( \sum_{j=1}^{r} \mathbf{e}_j^T \mathbf{K}_d^{-1} \mathbf{e}_j \right) + \frac{1}{t_2} \sum_{j=1}^{r} \text{Tr}a \left[ \Lambda_j(\tilde{X}) \right],
\]
where the (txt) matrix
\[
\Lambda_j(\tilde{X}) \triangleq (\mathbf{e}_j \otimes \mathbf{I}_t)^\dagger \left( (\mathbf{K}_d^{-1} \otimes \tilde{X}) \right)^{-1} (\mathbf{K}_d^{-1} \otimes \tilde{X}) (\mathbf{e}_j \otimes \mathbf{I}_t), \quad 1 \leq j \leq r,
\]
is semidefinite positive and Hermitian. Therefore, since our task becomes to minimize over $\tilde{X}$ the $r$ traces in the summation (B.1), we resort to an application of the Cauchy inequality \[13\] leading to the following lower bound:
\[
\text{Tr}a \left[ \Lambda_j(\tilde{X}) \right] \geq \frac{a^2 t}{t + a} \mathbf{I}_t, \quad 1 \leq j \leq r,
\]
thus we conclude that (16) is a necessary and sufficient condition for the optimality of $\tilde{X}$. ⋄

APPENDIX C - DERIVATION OF THROUGHPUT FORMULA IN (24)

Since for non singular MAI covariance matrix $\mathbf{K}_d$ the corresponding whitening filter
\[
\text{\overline{\mathbf{B}}} \triangleq (\mathbf{I} \otimes \mathbf{K}_d)^{-1/2} = \text{I}_{T_{pay}} \otimes \mathbf{K}_d^{-1/2},
\]
is a $(rT_{pay} \times rT_{pay})$ non singular block matrix, the resulting transformed observations\textsuperscript{12} $\overline{\mathbf{w}} \triangleq \text{\overline{\mathbf{B}}} \overline{\mathbf{y}}$ constitutes a sufficient statistic, so that the following equality holds:
\[
I(\overline{\mathbf{y}}; \overline{\mathbf{w}} | \overline{\mathbf{H}}) \equiv I(\overline{\mathbf{w}}; \overline{\mathbf{w}} | \overline{\mathbf{H}}) \triangleq h(\overline{\mathbf{w}} | \overline{\mathbf{H}}) - h(\overline{\mathbf{w}} | \overline{\mathbf{w}}, \overline{\mathbf{H}}),
\]
where $h(\cdot)$ denotes the differential entropy operator. Since from the assumed channel model (12) and linearity of the transformation induced by the whitening filter (C.1) it follows that the conditional r.v. $\overline{\mathbf{w}} | \overline{\mathbf{w}}, \overline{\mathbf{H}}$ is proper, Gaussian and with covariance matrix equal to
\[
\text{Cov}(\overline{\mathbf{w}} | \overline{\mathbf{w}}, \overline{\mathbf{H}}) = \text{I}_{rT_{pay}} + \frac{\sigma^2}{t} \left\{ \left( [\overline{\phi}(T_L + T_{tr} + 1) \ldots \overline{\phi}(T)]^T [\overline{\phi}^*(T_L + T_{tr} + 1) \ldots \overline{\phi}^*(T)] \right) \otimes \mathbf{K}_d^{-1} \right\},
\]
\textsuperscript{12}By applying the linear transformation (C.1) to the disturbance vector $\overline{\mathbf{d}}$ in (12) we arrive at the following relationship $\text{E} \{ \text{\overline{\mathbf{B}}} \overline{\mathbf{d}} (\text{\overline{\mathbf{B}}} \overline{\mathbf{d}})^\dagger \} = \text{I}_{rT_{pay}}$. So, according to a current taxonomy, we qualify as “spatial whitening filter” the matrix $\text{\overline{\mathbf{B}}}$ in (C.1).
we may directly express \( h(\vec{w} | \vec{\phi}, \hat{H}) \) in (C.2) as in
\[
h(\vec{w} | \vec{\phi}, \hat{H}) = rT_{\text{pay}} \log(\pi e) + \mathbb{E}_{\vec{\phi}} \left\{ \log \det \left( \mathbf{I}_{rt} + \frac{\sigma_z^2}{t} \left( \left( \mathbf{K}_d^{-1} \right)^* \otimes \left( \sum_{n=T_L+T_r+1}^{T} \phi(n) \phi(n)^* \right) \right) \right) \right\}. \tag{C.4}
\]
Now, although the pdf of the signal vector \( \vec{\phi} \) has been assumed Gaussian and proper, nevertheless for \( \sigma_z^2 > 0 \) the corresponding expectation in (C.4) resists to closed-form analytical evaluation even in the simplest case of spatially white MAI \cite[eqs.(8.95), (8.96)]{6}]. However, the Law of Large Numbers \cite[eqs.(8.95), (8.96)]{26} supports the conclusion that for large \( T_{\text{pay}} = T - T_L - T_r \) the summation in (C.4) converges (in the mean square sense) to the corresponding expectation \( T_{\text{pay}} \mathbf{R}_\vec{\phi} \), so that the following limit form holds for large \( T_{\text{pay}} \):
\[
h(\vec{w} | \vec{\phi}, \hat{H}) = rT_{\text{pay}} \log(\pi e) + \log \det \left( \mathbf{I}_{rt} + \frac{\sigma_z^2 T_{\text{pay}}}{t} \left( \mathbf{K}_d^{-1} \right)^* \otimes \mathbf{R}_\vec{\phi} \right), \tag{C.5}
\]
where \( \{P(m), 1 \leq n \leq t\} \) are the eigenvalues of input correlation matrix \( \mathbf{R}_\vec{\phi} \). Furthermore, due to the Gaussianity of disturbance in (12), the relationships (C.5), (C.6) hold regardless of values assumed by \( T_{\text{pay}} \) when \( \sigma_z^2 \) vanishes and/or the SINRs \( \{P(m)/\mu_l, 1 \leq m \leq t, 1 \leq l \leq r\} \) in (C.6) approach zero.

About the differential entropy \( h(\vec{w} | \hat{H}) \) in (C.2), for \( \sigma_z^2 > 0 \) it resists closed-form analytical evaluation even in the simplest case of \( r=t=1 \) and white MAI \cite[eqs.(8.95), (8.96)]{6}. However, since the covariance matrix of the r.v. \( \vec{w} | \hat{H} \) equates
\[
\mathbf{Cov}(\vec{w} | \hat{H}) = \mathbf{I}_{T_{\text{pay}}} \otimes \left[ \mathbf{I}_r + \sigma_z^2 P \mathbf{K}_d^{-1} + \frac{1}{t} \mathbf{K}_d^{-1/2} \hat{H} \mathbf{R}_\vec{\phi} \hat{H}^* \mathbf{K}_d^{-1/2} \right], \tag{C.7}
\]
thus, argumentations based on the Central Limit Theorem guarantees that the resulting bound
\[
h(\vec{w} | \hat{H}) \leq \log \left\{ (\pi e)^{rT_{\text{pay}}} \det \left[ \mathbf{Cov}(\vec{w} | \hat{H}) \right] \right\}, \tag{C.8}
\]
is satisfied as equality for large number \( t \) of transmit antennas. Furthermore, since for \( \sigma_z^2 \to 0 \) and/or vanishing SINRs \( \hat{H} \) converges to \( \mathbf{H} \) and then the r.v. \( \vec{w} | \hat{H} \) becomes proper and Gaussian, in these limit cases the upper bound (C.8) is reached regardless of value assumed by \( t \). Hence, after inserting (C.5) and (C.8) in (C.2), we directly arrive at (24). \( \diamond \)

**APPENDIX D - DERIVATION OF THE POWER ALLOCATION FORMULAS IN (28), (29)**
Similarly to [1,2], an application of the Hadamard inequality [12] to the relationship (24) followed by some standard matrix algebra allows us to develop the constrained sup in (23) as in

\[
C_G(H) = \sum_{m=1}^{r} \lg \left(1 + \frac{\sigma^2 P}{\mu_m}\right) + \\
\sup_{\sum_{m=1}^{t} P^*(m) \leq P_t} \left\{ f(P^*(1), ..., P^*(s)) - \frac{1}{T_{pay}} \sum_{m=s+1}^{r} \sum_{l=1}^{T} \lg \left(1 + \frac{\sigma^2 T_{pay} P^*(m)}{\mu_l}\right) \right\}, \tag{D.1}
\]

where

\[
f(P^*(1), ..., P^*(s)) = \sum_{m=1}^{s} f_m(P^*(m)) = \sum_{m=1}^{s} \left[ \lg(1 + \alpha_m P^*(m)) - \frac{1}{T_{pay}} \sum_{l=1}^{r} \lg(1 + \beta_l P^*(m)) \right]. \tag{D.2}
\]

is an additive objective function depending only on the powers to be allotted to the first \(s\) transmit antennas. Since the last two-fold summation into brackets in (D.1) vanishes only when \(P^*(s+1) = ... = P^*(t) = 0\), we can directly rewrite (D.1) as in

\[
C_G(H) = \sum_{m=1}^{r} \lg \left(1 + \frac{\sigma^2 P}{\mu_m}\right) + \sup_{\sum_{m=1}^{t} P^*(m) \leq P_t} \left\{ f(P^*(1), ..., P^*(s)) \right\}. \tag{D.3}
\]

Now, after posing \(\beta_{\max} \triangleq \max\{\beta_l, l = 1, ..., r\}\), we note that the second derivatives of the logarithmic functions \(f_m(P^*)\), \(1 \leq m \leq s\), in (D.2) are not positive over the region \(\mathcal{D}\) of \(\mathbb{R}^s\) given by

\[
\mathcal{D} \triangleq \left\{ (P^*(1), ..., P^*(s)) : P(m) \geq \max \left\{ 0, \frac{\beta_{\max} \sqrt{r} - \alpha_m \sqrt{T_{pay}}}{\alpha_m \beta_{\max} \sqrt{T_{pay} - r}} \right\}, m = 1, ..., s \right\}. \tag{D.4}
\]

Then, we conclude that the resulting sum-function \(f(P^*(1), ..., P^*(s))\), in (D.2) is guaranteed to be \(\cap\)–convex (at least) over \(\mathcal{D}\) that, in turn, approaches overall positive orthant \(\mathbb{R}^s_+\) of \(\mathbb{R}^s\) for vanishing \(\sigma^2\) and/or large \(T_{pay}\) (see eq. (27))\(^{13}\). Thus, an application of the Kuhn-Tucker conditions [14, eqs.(4.4.10), (4.4.11)] for the constrained maximization of the objective function (D.2) leads to the following relationships:

\[
(P^*(m) + \alpha_m^{-1})^{-1} - \frac{1}{T_{pay}} \sum_{l=1}^{r} (P^*(m) + \beta_l^{-1})^{-1} \leq 1/\rho, \text{ for all } m \text{ such that: } P^*(m) = 0, \tag{D.5}
\]

\[
(P^*(m) + \alpha_m^{-1})^{-1} - \frac{1}{T_{pay}} \sum_{l=1}^{r} (P^*(m) + \beta_l^{-1})^{-1} \leq 1/\rho, \text{ for all } m \text{ such that: } P^*(m) > 0. \tag{D.6}
\]

\(^{13}\)In practice, simple-to-test sufficient conditions guaranteeing the \(\cap\)–convexity of the objective function (D.2) over the overall orthant are the following ones:

\[
k_m \geq (\sigma^2 \sqrt{T_{pay}(\mu_m + P\sigma^2_t)})/(\mu_m \mu_{\min}), 1 \leq m \leq s,
\]

where \(\mu_{\min}\) denotes the minimum non zero eigenvalue of \(K_e\). In fact, when these last are met, all maxs in (D.4) vanish and \(\mathcal{D}\) approaches \(\mathbb{R}^s_+\). All carried out numerical tests of Sect.V were performed out under satisfaction of above sufficient conditions.
While the first Khun-Tucker condition (D.5) directly leads to (28), algebraic manipulations of the second one in (D.6) allow us to rewrite it in the following equivalent form:

\[
T_{\text{pay}} \left\{ \prod_{l=1}^{r} (P^*(m) + \beta_l^{-1}) \right\} \left[ \rho - (P^*(m) + \alpha_m^{-1}) \right] - \rho (P^*(m) + \alpha_m^{-1}) \left\{ \sum_{l=1}^{r} \prod_{j=1}^{r} (P^*(m) + \beta_j^{-1}) \right\} = 0,
\]

for all \( m \) such that: \( k_m^2 \geq (1 + \frac{\sigma^2_{\text{e}}}{\mu_m}) \left( \frac{t}{\rho} + \sigma^2_{\text{e}} \text{Tr} \left[ K_d^{-1} \right] \right) \). (D.7)

Eq.(D.7) is an \((r+1)\)-th order algebraic equation that, unfortunately, does not generally admit closed-form solution for the (unknown) optimized power level \( P^*(m) \). However, when all \( \{\beta_l, l = 1, ..., r\} \) in (27) share a same common value \( \beta_{\text{min}} \) so that the following condition is met:

\[
\beta_1 = \beta_2 = ... = \beta_r = \beta_{\text{min}},
\]

thus (D.7) reduces to the following 2nd-order algebraic equation:

\[
\beta_{\text{min}} P^*(m)^2 + \left\{ 1 - \beta_{\text{min}} \left[ \rho(1 - \frac{r}{T_{\text{pay}}} - \alpha_m^{-1}) \right] \right\} P^*(m) - \rho \alpha_m^{-1} (\alpha_m - \rho^{-1} - \frac{r \beta_{\text{min}} T_{\text{pay}}}{T_{\text{pay}}}) = 0,
\]

whose positive root is given by (29). Directly from the defining relationship (27) it follows that the above condition (D.8) is met when \( \sigma^2_{\text{e}} \) vanishes or/and \( T_{\text{pay}} \) is large and/or \( K_d \) is diagonal and/or all SINRs are low. Furthermore, when all above operating conditions fall short, the worst-case application scenario consists to assume the MAI covariance matrix \( K_d \) equating the diagonal one \( \mu_{\text{max}} I_r \), where \( \mu_{\text{max}} \) is the maximum eigenvalue of \( K_d \). Obviously, the optimized power level \( P^*(m) \) for the above mentioned worst-case scenario is still given by the positive root (29) of the algebraic equation (D.9). Thus, we may conclude that (29) may be interpreted in any case as the min-max solution for the constrained maximization of the objective function in (D.2) \( \diamond \).

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Fig. 1. Multi-Antenna System equipped with imperfect (forward) channel estimates $\hat{H}$ and impaired by MAI with spatial covariance matrix $K_d$.

Fig. 2. The packet structure (T $\triangleq T_L + T_{tr} + T_{pay}$)

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| PSEUDO-CODE FOR THE NUMERICAL IMPLEMENTATION OF THE PROPOSED OPTIMIZED POWER ALLOCATION ALGORITHM. |

| 1. Compute and order the eigenvalues of the MAI covariance matrix $K_d$; |
| 2. Compute the SVD of matrix $A$ in (26.1) and order its singular values; |
| 3. Set $P^*(m) = 0$, $1 \leq m \leq t$; |
| 4. Set $\rho = 0$ and $I(\rho) = \emptyset$; |
| 5. Set the step size $\Delta$; |
| 6. While $\left( \sum_{m \in I(\rho)} P^*(m) < P_t \right)$ do |
| 7. Update $\rho = \rho + \Delta$; |
| 8. Update the set $I(\rho)$ via eq. (30.1); |
| 9. Compute the powers $\{P^*(m), m \in I(\rho)\}$ via eq.(29); |
| 10. End; |
| 11. Compute the optimized powers $\{P^*(m), 1 \leq m \leq s\}$ via eqs. (28), (29); |
| 12. Compute the optimized shaping matrix $R_{\Phi}(opt.)$ via eq.(31); |
| 13. Compute the conveyed throughput $C_G(\hat{H})$ via eq.(32). |
Fig. 3. A general scheme of an ad-hoc network composed by (N+1) point-to-point links active over the same hot-spot area. Shaped arrows indicate the interfering links.

Fig. 4. A Hexagonal ad-hoc network with two interfering links.
Fig. 5. Sensitivity of the throughput $C_{T_G}$ conveyed by the reference link $T_r0 \rightarrow R_x0$ of Fig.4 on the squared error level $\sigma_r^2$ affecting the available channel estimation of the corresponding MIMO channel ($T_{pay} = 40$, $k=10$, $r=t=8$).

Fig. 6. Sensitivity of the throughput conveyed by the reference link $T_r0 \rightarrow R_x0$ of Fig.4 on the number $t=r$ of antennas ($T_{pay} = 80$, $k=10, \sigma_r^2 = 0.01$).
Fig. 7. Sensitivity of the throughput $C_G$ conveyed by the reference link $T_x,0 \rightarrow R_x,0$ of Fig.4 on the estimation errors affecting the available MAI covariance matrix ($T_{pay}=40$, $r=t=8$, $\sigma^2 = 0.015$).

Fig. 8. Throughput comparisons for the reference link $T_x,0 \rightarrow R_x,0$ of Fig.4 for $T_{pay}=80$, $k=1000$, $\sigma^2 = 0.1$. 
Fig. 9. Throughput comparison for the reference link $T_{d0} \rightarrow R_{d0}$ of Fig. 4 for $T_{PAY} = 80$, $r=t=10$, $k=0$, $\sigma^2 = 0.1$. 