

On the optimized download of large files

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Abstract

The emerging proxy-based wireless Content Delivery Networks (CDNs) should to be designed to download *huge-size* files over fading-affected channels. However, from a radio resource management point of view, several basic problems need to be still solved for such wireless delivery systems to operate efficiently. Specifically, due to the fading nature of the downlink channel, a still open basic problem is that to design optimal energy-allocation (e.g., scheduling) policies that *minimize* the requested download-time when constraints on the total available energy and peak-energy are simultaneously active. In this contribution, this problem is solved for application scenarios where the downlink slotted channel is *continuous-state*, and the resulting conveyed throughput is measured by *any desired* increasing concave rate-function. Specifically, the optimal energy-allocation policy minimizing the download-time is computed in closed-form and its performance is compared against that of a basic On-Off heuristic energy-scheduler for some Rayleigh-faded multi-antenna systems of practical interest. The carried-out performance comparisons point out that the presented optimal policy typically may outperform the heuristic one up to two-orders of magnitude, specially when the system is strongly energy-limited.

Index Terms

Minimum download time, radio resource management, energy-scheduling policies, CDNs, Multi-Antenna downlink channels, convex Calculus of Variations.

I. INTRODUCTION AND GOALS OF THE WORK

Due to widespread penetration of the broadband Internet access experienced in those last years, wireless multimedia service is getting increasingly popular among nomadic users and contributes a significant part of the current Internet traffic. Although media objects may be downloaded similar to usual texts and images via a download-and-play mode, nevertheless most users prefer to quickly start and then continuously playback a media object during the download (e.g., to use a streaming mode for the download). Hence, to reduce the user-experienced access delay as well as networking traffic, an effective means is to cache frequently requested media objects at proxies close to the end users. As a consequence, proxy caching is becoming the main component of all web-based multimedia distribution systems and CDNs [10,12,13,14]. In fact, due to the (quasi) static nature of their content and highly localized access interest, streaming over wireless channels of pre-stored multimedia contents could gain significant performance improvement from proxy caching. However, current proxies are generally designed to deliver conventional Web objects

(as, for example, HTML pages or GIF images), so that, they no still meet the hard QoS requirements of streaming applications.

Specifically, while the (average) size of a conventional Web-object is typically of the order of 1 – 100 Kbytes, a multimedia object exhibits high data-rates and long play-durations that give arise to file-sizes exceeding 700 Mbytes [10,11]. In addition, the streaming nature of the delivery demands for *fast download* of proxy-cached multimedia objects. In turn, due to the fadings affecting the download wireless channel, fast download may require *high* energy level to be radiated from the distribution node of the system. Thus, the right tuning of the tradeoff among radiated-energy and download-time may be no simple, specially in wireless application scenarios where the distribution nodes may be strongly energy-limited. For example, this may be the case of emerging satellite-based CDNs, where proxy-servers are located on satellite transponders equipped with *limited* energy-budget [1],[15, Chap.3],[24,Chap.6].

A. Related works

Overall, a main problem to be still solved in the sketched application scenarios is finding optimal energy-allocation scheduling policies for fast (e.g., minimum-time) download of huge-size data over fading channels when constraints on both overall available energy and allowed peak-energy are active. At the best of the authors' knowledge, up to date few contributions directly afforded this topic. Specifically, in [15, Sect.3.4] the problem has been tackled by modelling the fading downlink channel as a *discrete-state* link, and, then, by adopting a *linear* rate-function for measuring the resulting conveyed throughput. The more recent paper [5] refers to a broadcast application scenario where the channel-state is assumed *still discrete, no constraints* on the radiated energy are considered, and the size of the data to download *is limited*.

Barring these contributions, the previously mentioned energy-scheduling problem seems to be no still directly afforded in the technical literature. However, a number of papers have considered related problems, such that of the reduction of the waiting-time and energy-consumption in queue-based systems [1,2,3,4,8]. Also, several algorithms for wireless packet transmission have been developed in the last years [6,7,9] that rely on the processor-sharing model and provide varying levels of fairness. Although

those papers are, indeed, related to the afforded problem, they do not explicitly focus on the design of the optimal (e.g., time-minimizing) scheduler for energy-constrained wireless download of huge-size data, and, for the most part, those previous contributions assume a *discrete* channel-state model for describing the fading phenomena affecting downlink channels.

B. Main Contributions of this paper

Therefore, motivated by the above overview, in this paper we focus on the minimization of the average download-time of *huge-size* data over *continuous-state* slotted wireless channels, when system-constraints on the total energy available for the download and allowed peak-energy are present. The channel-state is assumed random and known to the transmitter on a per-slot basis, so that the transmitter may utilize this state-information for the optimal scheduling of the energy to be radiated. Specifically, the main contributions of this work may be so summarized.

- First, we develop both the optimal energy-allocation policy for the considered constrained optimization problem and the analytical tool for evaluating the corresponding performance. The optimality of the presented policy is retained over the *general class* of increasing concave rate-functions. Due to the continuous nature of the considered channel-state, the approaches followed in [5, 15] (based on the Dynamic Programming tool) fall short, so that we resort to some key-properties of the convex Calculus of Variations to resolve the considered optimization problem.
- Second, to gain insight about actual behavior of the optimal energy-allocation policy, we present the explicit forms assumed by this policy in correspondence of two (specific) rate-functions of practical interest, that is, the logarithmic and α -powered ones.
- Third, we test actual performance of the optimal policy on three Rayleigh-faded multi-antenna systems. Specifically, the first test-system we consider is a Single-Input Multiple-Output (SIMO) downloading system that employes a Maximum-Ratio-Combiner (MRC) at the receiver [16, Sect.5.3]. The second one is a Multiple-Input Single-Output (MISO) system equipped with an MRC at the transmitter [16, Sect.5.4]. The last one is a Multiple-Input Multiple-Output (MIMO) system utilizing Orthogonal Space-Time Block Codes (OSTBCs) to convey the data [16, Sect.6.3]. So doing, we

are able to evaluate the effects of transmit/receive diversity on the performance of the optimal energy-scheduling policy.

- Fourth, we compare the performance attained by the optimal policy against that of an On-Off type heuristic policy, so to evaluate the performance-gain offered by the presented optimal energy-scheduler.

C. Outline of the paper

The rest of this paper is organized as follows. After the system-modelling and problem setup of Sect.II, Sect.III presents the optimal energy allocation policy for the general class of concave increasing rate-functions. Thus, in Sect.IV we detail the forms assumed by the optimal policy in correspondence of two (specific) rate-functions of relevant interest, namely the logarithmic and α -powered rate-functions. Afterwards, in the first part of Sect.V we test actual performance of the optimal policy on above mentioned application scenarios. Thus, in the last part of Sect.V we compare the obtained performance with that achieved by the On-Off heuristic one and, then, we evaluate the resulting performance gain. Finally, some conclusions and hints are given in the final Sect.VI . Proofs of the main results are provided by the final Appendices.

Before proceeding, few words about the adopted notation. Bold capital letters denote matrices and underlined bold lower case symbols indicate vectors. Scalar random variables (r.v.s.) are denoted by bold (no underlined) lower case characters, while their outcomes are indicated by the corresponding (no bold) lower case symbols. $\text{Max}(a, b)$ and $\text{min}(a, b)$ report the maximum and minimum among the scalar a, b , while $\log(b)$ is the natural logarithm of b , $E\{\cdot\}$ is the expectation operator, \mathbb{R}_0^+ is the set of the nonnegative real numbers, \mathbb{R}^+ is the set of the strictly positive real ones, and \triangleq means "equal by definition". Finally, $P(\mathcal{A})$ is the probability of the event \mathcal{A} , $p(\sigma)$ is the probability density function (pdf) of the r.v. σ , while by the notation

$$E\{f(\sigma) | a \leq \sigma \leq b\} \triangleq \int_a^b f(\sigma)p(\sigma)d\sigma,$$

we mean the expectation of the function $f(\sigma)$ when the r.v. σ is limited to fall into the interval $[a,b]$.

II. SYSTEM MODEL AND PROBLEM SETUP

Let us consider a fading wireless downlink channel from a distribution node (base station, proxy-server, access-point, satellite transponder) to a (possibly mobile) user terminal. It is required that the distribution node transmits to the user terminal $\Delta \geq 1$ Information Units (IUs)¹. The downlink channel is assumed slotted², the slot starting at the (integer) time $t \geq 1$ is the slot t , and the fading effects impairing the channel are assumed constant over each slot (e.g., the so-called "block fading" model is assumed). To capture the random effects of the fading phenomena, the state of the channel $\sigma(t) \in \mathbb{R}_0^+$ over slot t is modelled as a real non-negative scalar r.v.³, and the corresponding random sequence $\{\sigma(t) \in \mathbb{R}_0^+, t \geq 1\}$ of the channel-state is assumed constituted by independent, identically distributed (i.i.d.) r.v.s sharing an (assigned) time-invariant probability density function⁴ $p(\sigma)$. Furthermore, we also assume that the value $\sigma(t)$ assumed by the channel state over slot t is perfectly known at the transmitter (e.g., at the distribution node) at the beginning of slot t ⁵.

Remark - About the validity limits of the assumed channel-model

Before proceeding, few words about the validity limits of the assumed channel-model are in order.

Technically speaking, the "block-fading" assumption holds when the coherence time of the downlink channel is over the time-duration of a slot. Since typical slot-times for downlink transmission are limited

¹Depending on the particular application scenario under consideration, IUs may be bits, nats, frames, packets, datagrams or web pages.

²Without loss of generality, the slot duration $T_{slot}(sec)$ is scaled up to the unit (e.g., $T_{slot} = 1$).

³Obviously, the meaning of the channel-state $\sigma(\cdot)$ is application depending. Without loss of generality and for sake of concreteness, we may consider $\sigma(t)$ be the Signal-to-Disturbance (e.g., the signal to noise-plus-interference) ratio measured at the output of the user receiver during slot t when the energy radiated by the distribution node is 1 (see Sect.V for more details on this point). The generalization of the presented results to the case of vector (e.g., multidimensional) channel-states does not look be direct and, currently, is under investigation by the authors.

⁴This pdf is assumed known to the transmitter. It describes (in a statistical sense) both fading phenomena and disturbances (noise plus interference) affecting the signal received by the user terminal.

⁵According to a quite common taxonomy, thereafter we refer to this assumption as that of Perfect Channel State Information (PCSI) at the transmitter.

up to 1-2 *ms* in current 3G/4G wireless distribution systems [23 and reference therein], in practice the "block-fading" assumption may be considered met for medium/low terminal speeds⁶.

About the i.i.d. assumption on the random sequence $\{\sigma(t)\}$ of the channel-states, this last may be assumed well fulfilled in TDMA, frequency-hopping or packet-based interleaved systems of practical interest, where each downloaded IU is independently detected by the user terminal [24 and references therein].

Due to the channel reciprocity, the assumption of PCSI of the transmitter may be considered reasonable in Time-Division-Duplexed (TDD) systems. It may be assumed well met also in 802.11-based 4GWLANS, where the RTS/CTS handshaking frames may be suitably employed to probe the downlink channel.

Finally, emerging Content Delivery systems for multimedia applications are *already equipped* with suitable control channels to forward back to the proxy-node the state of the downlink channel measured at the user terminal [13,14]. ■

A. The considered family of Rate-Functions

Let $\mathcal{E}(t)$ (*Joule*) be the energy radiated by the transmitter during slot t . Thus, it is reasonable to assume that the corresponding number $IU(t)$ of Information Units forwarded over the downlink channel over slot t depends *both* on $\mathcal{E}(t)$ and the channel-state $\sigma(t)$ via the rate-function $\mathcal{R}(\cdot; \cdot)$ adopted to measure the throughput-performance of the considered system, so that we can write

$$IU(t) = \mathcal{R}(\mathcal{E}(t); \sigma(t)), \quad t \geq 1. \quad (1)$$

Formally speaking, the rate-function $\mathcal{R} : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$ is a real non-negative function defined on $\mathbb{R}_0^+ \times \mathbb{R}_0^+$, that is measured in *IU/slot*. By fact, $\mathcal{R}(\cdot; \cdot)$ summarizes the throughput performance of the actually considered downlink transmission system, so that its behavior and analytical properties may depend on several system-related parameters, such as the requested QoS, the employed coding gain, the statistical feature of the fading and disturbing phenomena impairing the received signals, and so on (see

⁶For example, when the user terminals are of pedestrian mobility (e.g., up to 10 *Km/h*), the coherence time of the downlink channel is around 50 *ms*, so that the fading may be assumed "static" over slot-times of 1-2 *ms* [23].

the following Sect.IV for some examples of rate-functions of practical interest). Therefore, in the sequel we limit ourself to introduce few (quite mild) assumptions on $\mathcal{R}(\cdot; \cdot)$, that, by fact, are retained by all rate-functions of practical interest [21,22].

First, we assume that $\mathcal{R}(\mathcal{E}; \sigma)$ is continuous on $\mathbb{R}_0^+ \times \mathbb{R}_0^+$ and admits up to second-order continuous derivatives on $\mathbb{R}^+ \times \mathbb{R}_0^+$.

Second, we assume that $\mathcal{R}(\mathcal{E}; \sigma)$ vanishes at $\mathcal{E} = 0$ and $\sigma = 0$, e.g.,

$$\mathcal{R}(\mathcal{E} = 0; \sigma) \equiv \mathcal{R}(\mathcal{E}; \sigma = 0) \equiv 0. \quad (2)$$

Third, $\mathcal{R}(\mathcal{E}; \sigma)$ is assumed nondecreasing both for $\mathcal{E} \geq 0$ and $\sigma \geq 0$.

Fourth, for any assigned $\sigma \neq 0$, the function $\mathcal{R}(\mathcal{E}; \sigma)$ is assumed strictly concave in the \mathcal{E} variable, that is,

$$\mathcal{R}_{\mathcal{E}\mathcal{E}}(\mathcal{E}; \sigma) \triangleq \frac{\partial^2 \mathcal{R}(\mathcal{E}; \sigma)}{\partial \mathcal{E}^2} < 0, \quad \text{for } \mathcal{E} > 0 \text{ and } \sigma \neq 0. \quad (3)$$

Finally, we assume that the first-order derivative

$$\mathcal{R}_{\mathcal{E}}(\mathcal{E}; \sigma) \triangleq \frac{\partial \mathcal{R}(\mathcal{E}; \sigma)}{\partial \mathcal{E}}, \quad (4)$$

of the rate-function with respect to the \mathcal{E} -argument is nondecreasing for $\sigma \geq 0$.

B. Setup of the afforded Optimization Problem

For sake of readability, we list in Table I the main system-parameters involved by the optimization problem we go to afford. Roughly speaking, the optimization problem we focus on consists in the optimal allocation of the overall available energy \mathcal{E}_{tot} (*Joule*) over the downlink slots so to *minimize* the time T (in multiple of the slot time) requested to transfer an assigned number Δ of IUs, when an upper bound \mathcal{E}_{max} (*Joule*) on the allowed peak-energy per slot is *also* active. Since the download time T may be a critical system-performance parameter for large Δ ⁷, in the sequel we focus on application scenarios characterized by large Δ values. In this context, as in [15] it is reasonable to assume that the overall

⁷This is the case of proxy-based CDNs for multimedia applications, where the size Δ of an 1-hour MPEG1-coded video file cached at the proxy-server may be of the order of 670-700 Mbytes [10,11,13,14].

energy \mathcal{E}_{tot} available for the download is proportional to the size Δ of IUs to be transferred, so we may write

$$\mathcal{E}_{tot} = K\Delta, \quad (5)$$

where the (real non negative) K constant (in *Joule/IU*) dictates the average energy available for the download of a *single* IU⁸. In general, the energy $\mathcal{E}(t)$ radiated by the transmitter over slot t may depend on the channel-state $\sigma(t)$, the residual energy $\mathcal{E}^{(r)} \triangleq \left(\mathcal{E}_{tot} - \sum_{i=1}^{t-1} \mathcal{E}(i) \right)$ still available at the beginning of slot t , and also by the residual number $\Delta^{(r)} \triangleq \left(\Delta - \sum_{i=1}^{t-1} \Delta(i) \right)$ of IUs to be still downloaded at the beginning of slot t . So, this allows us to write

$$\mathcal{E}(t) \equiv \varepsilon \left(\sigma(t); \mathcal{E}^{(r)}(t); \Delta^{(r)}(t) \right), \quad t \geq 1, \quad (6)$$

where the energy allocation function $\varepsilon : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \times \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$ at the right-hand-side (r.h.s.) of (6) is a real non-negative function whose analytical form must be designed so to minimize the above mentioned download-time T . Therefore, since T is a r.v. whose outcomes also depend on the channel-states, the optimization problem we afford may be formally stated as in the following⁹:

$$\min_{\varepsilon \in \Omega} E\{T\} \quad (7)$$

$$s.t. : \sum_{i=1}^t \mathcal{E}(i) \leq K\Delta, \text{ for any } t \geq 1, \quad (7.1)$$

$$0 \leq \mathcal{E}(t) \leq \mathcal{E}_{max}, \text{ for any } t \geq 1, \quad (7.2)$$

where

$$\Omega \triangleq \left\{ \varepsilon \left(\sigma; \mathcal{E}^{(r)}; \Delta^{(r)} \right) \right\}, \quad (8)$$

is the set of the real non-negative functions defined on $\mathbb{R}_0^+ \times \mathbb{R}_0^+ \times \mathbb{R}_0^+$. The inequality in (7.1) accounts for the above mentioned constraint on the total available energy \mathcal{E}_{tot} (see eq.(5)), while the constraint in (7.2) limits the allowed peak-energy per slot. Since the minimization in (7) is over the set Ω of the

⁸This means that energy-limited systems are characterized by (very) low values of K .

⁹All the expectations are understood to be carry out over the pdf $p(\sigma)$ of the channel-state. Since (under the above reported assumptions) the considered transmission system is stationary, all the expectations involved by our analysis do no depend on the time-index t .

energy allocation *functions* $\varepsilon(\cdot; \cdot; \cdot)$, the considered optimization problem is an instance of Calculus of Variations [18]. The convex property retained by this problem clearly stands up by re-casting it in a suitable *equivalent* form. Specifically, let us

$$r \triangleq \mathbb{E}\{IU(t)\} \equiv \overline{IU}, \quad (IU/\text{slot}) \quad (9)$$

be the average number of IUs transferred over the downlink channel during a slot time. Since, by definition, r in (9) equates the ratio: $\Delta/\mathbb{E}\{T\}$, minimizing $\mathbb{E}\{T\}$ is equivalent to maximize r . So, the constrained optimization problem in (7) may be equivalently re-written as in

$$\max_{\mathcal{E} \in \Omega} \mathbb{E}\{IU(t)\}, \quad (10)$$

$$s.t. : \sum_{i=1}^t \mathcal{E}(i) \leq K\Delta, \text{ for any } t \geq 1, \quad (10.1)$$

$$0 \leq \mathcal{E}(t) \leq \mathcal{E}_{max}, \text{ for any } t \geq 1. \quad (10.2)$$

Now, since the rate-function in (1) is concave over $\mathcal{E} \geq 0$, and the expectation operator preserves concavity, the objective function in (10) is concave over $\varepsilon \in \Omega$. Finally, since both constraints (10.1), (10.2) are convex, we conclude that the constrained optimization problem in (10) is an instance of *convex* constrained Calculus of Variations.

C. Problem re-statement for large data to download

For large Δ (e.g., for $\Delta \rightarrow \infty$), a suitable application of the (strong) law of large numbers allows us to re-state the constrained optimization problem in (10) into an *equivalent (but simpler)* form. In fact, after indicating as

$$\mathcal{A} \triangleq \{\varepsilon(\sigma) \in \mathbb{R}_0^+\} \subset \Omega, \quad (11)$$

the subset of Ω constituted by the real non-negative energy allocation function $\varepsilon : \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$ depending *only* on the state σ of the downlink channel, in the Appendix I is proved that the following *Proposition 1* holds for large Δ :

Proposition 1: (Problem Restatement for large Δ)

Let us consider met the previously reported assumptions on the system model and rate-function. Thus, for large Δ , the constrained optimization problem in (10) assumes the following *equivalent* form:

$$\max_{\varepsilon(\sigma) \in \mathcal{A}} \int_0^\infty \mathcal{R}(\varepsilon(\sigma); \sigma) p(\sigma) d\sigma, \quad (12)$$

$$s.t. : \int_0^\infty \varepsilon(\sigma) p(\sigma) d\sigma \leq K \int_0^\infty \mathcal{R}(\varepsilon(\sigma); \sigma) p(\sigma) d\sigma, \quad (12.1)$$

$$0 \leq \varepsilon(\sigma) \leq \mathcal{E}_{max}, \text{ for any } \sigma \geq 0. \quad (12.2)$$

■

Essentially, *Proposition 1* guarantees that, for large Δ , the energy allocation policy $\varepsilon(\sigma; \mathcal{E}^{(r)}; \Delta^{(r)}) \in \Omega$ achieving the (constrained) maximum in (10) *does no longer* depend on the residual available energy $\mathcal{E}^{(r)}$ and residual data $\Delta^{(r)}$ to download, but it depends *only* on the channel-state σ . In this respect, the above *Proposition 1* generalizes a result previously reported in [15, p.90] for the *particular* case of downlink channels with *finite* number of states and throughput-performance measured via *linear* rate-functions (that is, $\mathcal{R} \equiv \sigma \mathcal{E}$).

III. THE OPTIMAL ENERGY ALLOCATION POLICY FOR LARGE DATA TO DOWNLOAD

After denoting by $\varepsilon_0(\sigma)$ the *optimal* energy allocation policy achieving the constrained maximum in (12), let us indicate by

$$r_0 \triangleq \int_0^\infty \mathcal{R}(\varepsilon_0(\sigma); \sigma) p(\sigma) d\sigma, \quad (IU/slot) \quad (13)$$

and

$$T_0 \triangleq \frac{\Delta}{r_0}, \quad (slot) \quad (14)$$

the average number r_0 of IUs downloaded over a slot-time and the corresponding download-time T_0 (in multiple of the slot-time)¹⁰. Thus, since the constant K in (5) fixes the available average energy per IU, it is reasonable to conjecture that the form of the optimal policy $\varepsilon_0(\sigma)$ strongly depends on the value actually assumed by K . This conjecture is, indeed, true, as detailed by the analysis reported in the following sub-Sections for the three cases of large, small and intermediate K values.

¹⁰Since T_0 is evaluated for the optimal energy allocation policy $\varepsilon_0(\sigma)$, thus, by definition, T_0 represents the *minimum* average download-time allowed the considered system.

A. Case of Large K values

Roughly speaking, for large K the constraint in (12.1) becomes no active, and thus the transmitter may radiate at the peak energy \mathcal{E}_{max} , *regardless* of the value σ assumed by the channel-state. Specifically, this transmit policy is feasible when the following condition is met (see eq.(12.1)):

$$\int_0^\infty \mathcal{E}_{max} p(\sigma) d\sigma \leq K \int_0^\infty \mathcal{R}(\mathcal{E}_{max}; \sigma) p(\sigma) d\sigma. \quad (15)$$

Therefore, after indicating by

$$r_{max} \triangleq \int_0^\infty \mathcal{R}(\mathcal{E}_{max}; \sigma) p(\sigma) d\sigma, \quad (IU/slot) \quad (16)$$

the ultimate (e.g., the maximum) average rate sustainable by the considered system under the peak-energy constraint (12.2), the following result directly arises.

Proposition 2: (The Optimal Energy allocation policy for large K values)

Under the above reported assumptions about the considered system, for

$$K \geq K_H \triangleq \frac{\mathcal{E}_{max}}{r_{max}}, \quad (Joule/IU), \quad (17)$$

the optimal energy allocation policy achieving the constrained maximum in (12) is

$$\varepsilon_0(\sigma) = \mathcal{E}_{max}, \quad \text{for any } \sigma \geq 0. \quad (18)$$

■

B. Case of Small K values

It is reasonable to believe that, for small K values, the available total energy in (5) may be no sufficient to allow the download of the *overall* assigned IUs. The following *Proposition 3* (proved in the Appendix II) confirms, indeed, this conjecture.

Proposition 3: (The Optimal Energy allocation policy for small K values)

After indicating by

$$m(\sigma) \triangleq \lim_{\mathcal{E} \rightarrow 0^+} \mathcal{R}_\varepsilon(\mathcal{E}; \sigma), \quad (19)$$

the \mathcal{E} -derivative of the rate-function evaluated at $\mathcal{E} = 0$, let us introduce the following lower threshold on K :

$$K_L \triangleq \frac{1}{\sup_{\sigma \geq 0} \{m(\sigma)\}}. \quad (20)$$

Thus, for $K \leq K_L$, the optimal energy allocation policy becomes¹¹

$$\varepsilon_0(\sigma) = 0, \text{ almost surely (a.s.) for } \sigma \geq 0. \quad (21)$$

■

About the above results, some explicative comments are in order. First, since the rate-function has been assumed no-decreasing in the \mathcal{E} -variable, the derivative in (19) is no negative for $\sigma \geq 0$, so that the K_L constant in (20) is guaranteed to be no negative. Second, by exploiting the concavity property of the rate-function [17, p.70], it can be proved that the threshold K_L in (20) *does not exceed* the corresponding threshold K_H in (17) (e.g., $K_L \leq K_H$). Third, we observe that, under the allocation policy (21), the average rate r_0 in (13) vanishes, so that the corresponding download-time T_0 in (14) becomes infinite. This means that, for $K \leq K_L$, the available average energy per slot *does not allow* the download of the *overall* assigned Δ IUs, *regardless* of the adopted feasible energy allocation policy. This is, indeed, the engineeristic meaning of the result reported by the *Proposition 3*.

C. Case of Intermediate K values

For K falling in the (open) interval $]K_L, K_H[$, the convex optimization problem in (12) satisfies the Slater's conditions (see the Appendix III), so that the constrained maximum in (12) equates the constrained minimum of the associated dual-problem [17, Chap.5], [18, Chap.2],

$$\min_{\mu \geq 0} g(\mu), \quad (22)$$

$$s.t. : \mu \in \text{Dom}(g) \triangleq \{\mu \in \mathbb{R}_0^+ : |g(\mu)| < \infty\}. \quad (22.1)$$

Specifically, the scalar non-negative μ parameter in (22) is the dual-variable of the primal-problem in (12) [17, Chap.5], [18, Chap.2], while the real non-negative *convex* function $g(\mu) : \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$ is the corresponding dual-function. This last is defined as in [17, Chap.5], [18, Chap.2]

$$g(\mu) \triangleq \max_{0 \leq \varepsilon(\sigma; \mu) \leq \mathcal{E}_{max}} \int_0^\infty F(\sigma; \varepsilon(\sigma; \mu); \mu) d\sigma, \quad (23)$$

¹¹By definition, "almost surely" means that the set of channel-states where $\varepsilon_0(\sigma) > 0$ is with null measure, that is:

$$\int_{\sigma: \varepsilon_0(\sigma) > 0} p(\sigma) d\sigma = 0.$$

where the non-negative dummy-function $F : \mathbb{R}_0^+ \times \mathbb{R}_0^+ \times \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$ is given by¹²

$$F(\sigma; \varepsilon(\sigma; \mu); \mu) \triangleq p(\sigma) [(1 + K\mu)\mathcal{R}(\varepsilon(\sigma; \mu); \sigma) - \mu\varepsilon(\sigma; \mu)]. \quad (24)$$

Now, since $F(\cdot; \cdot; \cdot)$ is concave in the $\varepsilon(\cdot; \cdot)$ variable, thus, for any assigned μ , the energy allocation policy $\varepsilon^*(\sigma; \mu)$ achieving the *unconstrained* (e.g., no limited up to \mathcal{E}_{max}) maximum of $F(\cdot; \cdot; \cdot)$ may be computed by solving with respect to $\varepsilon(\cdot; \cdot)$ the associated (Euler) equation:

$$F_\varepsilon(\sigma; \varepsilon(\sigma; \mu); \mu) = 0, \quad \text{for any } \sigma \geq 0, \quad (25)$$

where $F_\varepsilon(\cdot; \cdot; \cdot) \triangleq \frac{\partial}{\partial \varepsilon} F(\cdot; \cdot; \cdot)$. To proceed, let us also define the following μ -depending lower and upper thresholds on the channel-state values:

$$\sigma_L(\mu) \text{ is the biggest } \sigma \geq 0 \text{ such that: } \mathcal{R}_\varepsilon(\mathcal{E}; \sigma) < \mu/(1 + K\mu), \text{ for any } \mathcal{E} \geq 0, \quad (26)$$

and

$$\sigma_H(\mu) \text{ is the smallest } \sigma \geq 0 \text{ such that: } \mathcal{R}_\varepsilon(\mathcal{E}; \sigma) > \mu/(1 + K\mu), \text{ for any } \mathcal{E} \geq 0, \quad (27)$$

where $\mathcal{R}_\varepsilon(\cdot; \cdot)$ is the \mathcal{E} -derivative in (4) of the rate-function. Thus, in the Appendix IV it is proved that, for K falling in the (open) interval $]K_L, K_H[$, the optimal energy allocation policy assumes the form detailed by the following *Proposition 4*:

Proposition 4: (The Optimal Energy Allocation Policy for intermediate K values)

Under the above reported assumptions on the considered system, for $K \in]K_L, K_H[$ the optimal energy allocation policy is given by

$$\varepsilon_0(\sigma) \equiv \varepsilon_0(\sigma; \mu^*) = \begin{cases} 0, & \text{for } \sigma \leq \sigma_L(\mu^*), \\ \max\{0; \min\{\varepsilon^*(\sigma; \mu^*); \mathcal{E}_{max}\}\}, & \text{for } \sigma \in]\sigma_L(\mu^*), \sigma_H(\mu^*)[, \\ \mathcal{E}_{max}, & \text{for } \sigma \geq \sigma_H(\mu^*). \end{cases} \quad (28)$$

Furthermore, the μ^* parameter in (28) is the optimal value of the dual-variable, that may be computed by solving (with respect μ) the following (algebraic) equation:

$$\int_{\sigma_L(\mu)}^{+\infty} F_\mu(\sigma; \varepsilon_0(\sigma; \mu); \mu) d\sigma = 0, \quad (29)$$

¹²For sake of clearness, thereafter we adopt the (somewhat redundant) notation $\varepsilon(\sigma; \mu)$ to stress that the energy allocation policies computed on the basis of the dual-function in (23) also depend on the value assumed by the dual-variable μ .

with $F_\mu(\cdot; \cdot; \cdot)$ being the μ -derivative of the F function in (24). ■

The performance attained by the optimal policy in (28) is directly computable via the above reported relationships (13),(14). In order to actually compute the optimal policy (28), the following Remarks may be of interest.

Remark 1 - On the computation of the $\varepsilon^(\cdot; \cdot)$ policy*

By performing the \mathcal{E} -derivative of the F function in (24), it can be viewed that the solution $\varepsilon^*(\cdot; \cdot)$ of the (functional) equation in (25) may be computed *in closed-form* as in

$$\varepsilon^*(\sigma; \mu) = \mathcal{R}_\varepsilon^{-1} \left(y = \frac{\mu}{1 + K\mu}; \sigma \right), \quad (30)$$

where $\mathcal{R}_\varepsilon^{-1}(y; \sigma)$ denotes the inverse function of $\mathcal{R}_\varepsilon(\mathcal{E}; \sigma)$ in (4) done with respect to the \mathcal{E} -variable¹³

Therefore, the μ -derivative of (30) equates

$$\frac{\partial \varepsilon^*(\sigma; \mu)}{\partial \mu} = \left(\frac{1}{1 + K\mu} \right)^2 \left[\frac{\partial}{\partial y} \mathcal{R}_\varepsilon^{-1}(y; \sigma) \right]_{y=\mu/(1+K\mu)} \quad (31)$$

Remark 2 - On the computation of $F_\mu(\cdot; \cdot; \cdot)$

Directly from the definition in (24), the μ -derivative in (29) of the F function may be computed as in

$$F_\mu(\sigma; \varepsilon_0(\sigma; \mu); \mu) = p(\sigma) [K\mathcal{R}(\varepsilon_0(\sigma; \mu); \mu) - \varepsilon_0(\sigma; \mu)] + \frac{\partial \varepsilon_0(\sigma; \mu)}{\partial \mu} F_\varepsilon(\sigma; \varepsilon_0(\sigma; \mu); \mu). \quad (32)$$

Furthermore, from (28) we also have

$$\frac{\partial \varepsilon_0(\sigma; \mu)}{\partial \mu} = \frac{\partial \varepsilon^*(\sigma; \mu)}{\partial \mu} \{[(\mathcal{E}_{max} + \varepsilon^*(\sigma; \mu)) \delta(\mathcal{E}_{max} - \varepsilon^*(\sigma; \mu)) + U(\mathcal{E}_{max} - \varepsilon^*(\sigma; \mu))] U(\varepsilon^*(\sigma; \mu))\}, \quad (33)$$

where $\delta(\cdot)$ is the Dirac-pulse, $U(\cdot)$ is the Heaviside (e.g., the unit-step) function, and $\partial \varepsilon^*(\sigma; \mu)/\partial \mu$ is given by (31). ■

¹³Under the assumptions on $\mathcal{R}_\varepsilon(\cdot; \cdot)$ previously reported in Sect.II-A, Dini's theorem holds. Hence, the existence of the inverse function (30) is guaranteed over the domain of interest.

Remark 3 - Constrained and Unconstrained energy allocation policies

We conclude this sub-Section by explicitly noting that, when the optimal policy in (28) satisfies the following relationship:

$$\varepsilon_0(\sigma; \mu) \equiv \varepsilon^*(\sigma; \mu), \quad \text{for any } \sigma \in (\sigma_L(\mu), \sigma_H(\mu)), \quad (34)$$

thus the derivative $F_\varepsilon(\cdot; \cdot; \cdot)$ in (32) *vanishes* over the overall interval $\sigma \in (\sigma_L(\mu), \sigma_H(\mu))$. We anticipate that this is the case of the logarithmic and α -powered rate-functions considered in the following Sect.IV.

■

D. The Overall Optimal Energy Allocation Policy

Table II shortly summarizes the main steps needed to actually compute the overall optimal energy allocation policy $\varepsilon_0(\cdot; \cdot)$ and the related performance indices r_0, T_0 . From the computation procedure reported in Table II, some considerations about the implementation-complexity and robustness of the optimum energy allocation policy may be drawn.

Remark 1 - On the implementation-complexity of the optimal policy

The optimal value μ^* of the dual-variable and the thresholds $K_L, K_H, \sigma_L, \sigma_H$ may be computed off-line on the basis of the system-parameters and pdf $p(\sigma)$ of the channel-state. Thus, the on-line implementation of the optimal policy $\varepsilon(\cdot; \cdot)$ may be directly accomplished via a three-way threshold detector, whose input is the value assumed (slot-by-slot) by the channel-state σ (see eq.(28)).

■

Remark 2 - On the robustness of the optimal policy

About the robustness of $\varepsilon_0(\cdot; \cdot)$ against the effects of the channel-state pdf, Table II points out that the overall form assumed by $\varepsilon_0(\cdot; \cdot)$ *does not depend* on the adopted channel-state pdf (see eqs.(18), (21), (28)). By fact, an examination of eqs. (29), (32) shows that *only* the optimal value μ^* assumed by the dual-variable for $K \in]K_L, K_H[$ *depends*, indeed, on this pdf. Thus, we may conclude that the optimal policy $\varepsilon_0(\cdot; \cdot)$ is *robust* against the effects induced by the pdf adopted for describing the statistical behavior of the channel-states.

■

IV. FORMS OF THE OPTIMAL POLICY FOR SOME RATE-FUNCTIONS OF PRACTICAL INTEREST

The expression previously reported in Sect.III for the optimal policy $\varepsilon_0(.,.)$ are general and hold for any rate-function satisfying the assumptions of Sect.II-A. However, to gain insight into the performance improvements actually offered by the optimal policy, it may be suitable to specialize these general expressions to some rate-functions of practical interest.

A. Case of the logarithmic rate-function

Specifically, the first rate-function we consider is the logarithmic one, defined by the (usual) relationship

$$\mathcal{R}(\mathcal{E}; \sigma) \equiv \log(1 + \mathcal{E}\sigma), \quad (\text{nats/slot}). \quad (35)$$

As it is known, this last measures (possibly, within up to a scaling factor) the so-called Shannon's capacity of several transmission systems of large interest, as, for example, those considered in the following Sect.V. Thus, after introducing (35) into the (general) expressions (17), (20), (26), (27), we obtain for the thresholds $K_L, K_H, \sigma_L(\mu^*)$ and $\sigma_H(\mu^*)$ the explicit expressions reported in the first four rows of Table III. Furthermore, since it can be viewed that the relationship (34) holds for the logarithmic function (35), thus the general expression in (28) assumed by the optimal policy for $K \in]K_L, K_H[$ becomes

$$\varepsilon_0(\sigma) \equiv \varepsilon_0(\sigma; \mu^*) = \begin{cases} 0, & \text{for } \sigma \leq \sigma_L(\mu^*), \\ (1/\sigma_L(\mu^*) - 1/\sigma) & \text{for } \sigma_L(\mu^*) < \sigma < \sigma_H(\mu^*), \\ \mathcal{E}_{max}, & \text{for } \sigma \geq \sigma_H(\mu^*). \end{cases} \quad (36)$$

Finally, the (algebraic) equation (29) for the computation of the optimal value μ^* of the dual variable μ takes on following form:

$$\left[K \log \left(K + \frac{1}{\mu} \right) - \left(K + \frac{1}{\mu} \right) \right] P(\sigma_L(\mu) \leq \sigma \leq \sigma_H(\mu)) + E \left\{ K \log \sigma + \frac{1}{\sigma} \middle| \sigma_L(\mu) \leq \sigma \leq \sigma_H(\mu) \right\} \equiv \mathcal{E}_{max} P(\sigma \geq \sigma_H(\mu)) - K E \{ \log(1 + \sigma \mathcal{E}_{max}) \mid \sigma \geq \sigma_H(\mu) \}, \quad (37)$$

where the explicit expression assumed by the probabilities and conditional expectations involved by (37) are detailed in the last four rows of Table III.

B. Case of α -powered rate-functions

The so called α -powered rate-function is defined by

$$\mathcal{R}(\mathcal{E}; \sigma) \equiv (\sigma \mathcal{E})^\alpha, \quad (IU/slot) \quad (38)$$

with the α exponent falling into the (open) interval $(0, 1)$. This rate-function is sometime utilized to measure the throughput sustained by networking and transport protocols [7,15,17]. By inserting (38) into (17), (20), (26), (27), we arrive at the explicit expressions reported in the Table IV for the thresholds $K_L, K_H, \sigma_L(\mu^*)$ and $\sigma_H(\mu^*)$. Furthermore, the relationship (34) is still verified also by the rate-function (38), so that, in this case, the general expression (28) reduces to the following one:

$$\varepsilon(\sigma) \equiv \varepsilon(\sigma; \mu^*) = \begin{cases} \mathcal{E}_{max} [\alpha \sigma^\alpha (K + 1/\mu^*)]^{\frac{1}{1-\alpha}} & \text{for } 0 \leq \sigma \leq \sigma_H(\mu^*) \\ \mathcal{E}_{max}, & \text{for } \sigma > \sigma_H(\mu^*). \end{cases} \quad (39)$$

Lastly, the (algebraic) equation (29) for the computation of μ^* becomes

$$\begin{aligned} & \left[\alpha \left(K + \frac{1}{\mu} \right) \right]^{\alpha/1-\alpha} \left[K - \alpha \left(K + \frac{1}{\mu} \right) \right] \mathbb{E} \left\{ \sigma^{\frac{\alpha}{1-\alpha}} \mid \sigma \leq \sigma_H(\mu) \right\} - \\ & - \mathcal{E}_{max} P(\sigma \geq \sigma_H(\mu)) + K \mathcal{E}_{max}^\alpha \mathbb{E} \{ \sigma^\alpha \mid \sigma \geq \sigma_H(\mu) \} = 0. \end{aligned} \quad (40)$$

Remark - The limit case of $\alpha = 1$

For $\alpha = 1$ the rate function (38) reduces to the linear one: $\mathcal{R}(\mathcal{E}; \sigma) \equiv \sigma \mathcal{E}$, already considered in [15, Chap.3]. Now, the limit expression assumed by $\sigma_H(\mu^*)$ in Table IV is $\lim_{\alpha \rightarrow 1} \sigma_H(\mu^*) = (K + 1/\mu^*)^{-1}$, and the corresponding limit of the optimal policy in (39) for $\sigma \in [0, \sigma_H(\mu^*)]$ vanishes. Hence, for $\alpha = 1$ the optimal policy (39) reduces to the following On-Off one:

$$\varepsilon_0(\sigma) \equiv \varepsilon_0(\sigma; \mu^*) = \begin{cases} 0, & \text{for } 0 \leq \sigma \leq (K + 1/\mu^*)^{-1} \\ \mathcal{E}_{max}, & \text{for } \sigma > (K + 1/\mu^*)^{-1}, \end{cases} \quad (41)$$

where μ^* is the solution (in the μ -variable) of the algebraic equation

$$\int_{(K+\frac{1}{\mu})^{-1}}^{\infty} \sigma p(\sigma) d\sigma = \frac{1}{K} \int_{(K+\frac{1}{\mu})^{-1}}^{\infty} p(\sigma) d\sigma,$$

obtained by passing to the limit for $\alpha \rightarrow 1$ the relationship (40). Since the optimality of the On-Off energy allocation policy for the linear rate-function has been already proved in [15, Sect.3.4] for the case

of *discrete-state* downlink channels, we conclude that this optimality property is *still retained* also in the case of downlink channels with *continuous* (e.g., uncountable and infinite) states. ■

V. APPLICATION EXAMPLES AND PERFORMANCE COMPARISONS

In this Section we test actual performance of the optimal energy allocation policy on three Rayleigh-faded application scenarios of practical interest. Specifically, we consider,

- (i) Single-Input Multiple-Output (SIMO) downloading systems, where the receiver is equipped with a Maximal-Ratio Combiner (MRC) [16, Sect.5.3].
- (ii) Multiple-Input Single-Output (MISO) downloading systems, where the transmitter is equipped with an MRC [16, Sect.5.4].
- (iii) Multiple-Input Multiple-Output (MIMO) downloading systems employing Orthogonal Space-Time Block Codes (OSTBC) [16, Sect.6.3].

Since the Shannon's capacity of all above mentioned systems is given (possibly, within up to a scaling factor) by the logarithmic rate-function in (35) [16, Sects.5.3,5.4,6.3], all the numerical results we present in this Section refer to this rate-function. Furthermore, as already anticipated in the Note 3, in all the considered application scenarios the channel-state σ represents the instantaneous (e.g., fading affected) Signal-to-Noise Ratio (SNR) measured at the output of the system-receiver when the energy radiated by the corresponding transmitter over a slot-time equates the unit.

A. A benchmark energy allocation policy

To evaluate the performance gain achieved by the optimal energy allocation policy (36), in the following sub-Sections we compare its performance against that of the simple On-Off heuristic policy $\varepsilon_{HE}(\sigma)$ defined by

$$\varepsilon_{HE}(\sigma) \triangleq \begin{cases} 0, & \text{for } \sigma < Th^*, \\ \mathcal{E}_{max}, & \text{for } \sigma \geq Th^*, \end{cases} \quad (42)$$

where the threshold Th^* in (42) is set so to satisfy the following energy constraint¹⁴:

$$\mathcal{E}_{max} \int_{Th^*}^{+\infty} p(\sigma) d(\sigma) = K \int_{Th^*}^{+\infty} \log(1 + \sigma \mathcal{E}_{max}) p(\sigma) d\sigma. \quad (43)$$

Thus, the average download rate r_{HE} (*nat/slot*) of the heuristic policy (42) equates

$$r_{HE} \triangleq \int_{Th^*}^{\infty} \log(1 + \sigma \mathcal{E}_{max}) p(\sigma) d\sigma \equiv \frac{\mathcal{E}_{max}}{K} \int_{Th^*}^{+\infty} p(\sigma) d\sigma, \quad (\textit{nats/slot}) \quad (44)$$

so that the corresponding average download-time T_{HE} is given by

$$T_{HE} = \Delta / r_{HE}. \quad (\textit{slot}) \quad (45)$$

About implementation-complexity, we note that the on-line effort requested to implement the heuristic policy (42) is, by fact, almost the same needed for the optimal policy (36), because both policies may be implemented, indeed, via (simple) threshold detectors.

B. Performance of Rayleigh-faded SIMO Systems

As a first test system, let us consider the flat-faded downlink channel going from a transmitter T_X equipped with $N_t = 1$ antenna to a receiver R_X (e.g., an user terminal) equipped with $N_r \geq 1$ receive antennas. As in [16, Sect.5.3], let us assume that the channel coefficients $\{h_j \in \mathbb{C}, j = 1, \dots, N_r\}$ linking the transmitter to the N_r receive antennas are mutually independent, zero-mean, unit-variance, Gaussian distributed r.v.s, whose values are known to the receiver. Thus, the Shannon' capacity C_{R-MRC} (*nats/slot*) of the overall channel from the transmitter input to the output of the MRC receiver equates [16, Sect.5.3]

$$C_{R-MRC} = \log \left(1 + \frac{\mathcal{E}}{\mathcal{N}_0} \left(\sum_{j=1}^{N_r} |h_j|^2 \right) \right), \quad (\textit{nats/slot}) \quad (46)$$

where \mathcal{E} (*Joule*) is the energy radiated by the transmitter over a slot-time, and \mathcal{N}_0 (*Joule*) is the two-side power density spectrum of the Additive White Gaussian Noise (AWGN) measured at the output of each

¹⁴So doing, the policy (42) satisfies both the energy constraints (12.1), (12.2). Furthermore, (43) guarantees that the policy (42) attains the r.h.s. of the constraint (12.1). Hence, the performance in (44), (45) attained by the heuristic policy is the *best* one achievable by (42) under constraints (12.1), (12.2).

receive antenna. Thus, the channel-state $\sigma \triangleq \frac{1}{N_0} \left(\sum_{j=1}^{N_r} |h_j|^2 \right)$ of the considered system is a central chi-square r.v. with $2N_r$ degrees of freedom, whose pdf equates [19, p.26]

$$p(\sigma) = \frac{(\mathcal{N}_0)^{N_r}}{(N_r - 1)!} \sigma^{N_r-1} \exp(-\mathcal{N}_0\sigma), \quad \sigma \geq 0. \quad (47)$$

The optimal energy allocation policy $\varepsilon_0(\sigma)$ and the associated performance indices r_0, T_0 are obtained by evaluating eqs. (36), (37), (13), (14) for the pdf in (47)¹⁵. The resulting behavior of the optimal policy (36) is reported in Fig.1 for the case of $N_r = 1, \mathcal{N}_0 = 1(\text{Joule}), \mathcal{E}_{max} = 10(\text{Joule})$, and various values of K . It may be noted that the behaviour of $\varepsilon_0(\cdot)$ in (36) becomes steeper for increasing K values, so to approach an On-Off behavior for $K \gg 1$. Fig.2 points out the key-role played by the constant $K(\text{Joule/nat})$ on the average download-times T_0, T_{HE} of the optimal and heuristic policies. Specifically, the numerical plots of Fig.2 show that the ratio T_{HE}/T_0 is around 10^5 for small K values (e.g., for K around $2(\text{Joule/nat})$), while it reduces to about 3.5 at medium K values (we say, for K around $3(\text{Joule/nat})$), and then it approaches the unit for $K \geq 3.7(\text{Joule/nat})$. Thus, the first conclusion we may draw is that the performance improvement offered by the optimal policy is substantial at medium/low K values (e.g., for K falling in the interval $[2, 3.5](\text{Joule/nat})$).

The effects of the number N_r of the receive antennas are stressed by the plots of Fig.3. Specifically, Fig.3 points out that, although performance of both the optimal and heuristic policies improve for growing N_r , nevertheless the ratio r_0/r_{HE} among the corresponding average information rates is *over 9.5* for N_r values up to 8. This conclusion is confirmed by the plots of Fig.4, that point out that the ratio T_{HE}/T_0 among the corresponding average download-times scales from about 10^5 at $N_r = 1$ to 20 at $N_r = 8$. Hence, the second conclusion we may draw is that the performance gain offered by the optimal policy in SIMO MRC-based downloading applications may be substantial for values N_r of the receive antennas limited up to 8-9.

¹⁵For computing the involved integrals, we resorted to [25, no.3.353.5, 2.321.2, 4.337.2]. For sake of brevity, we omitted the related analytical details.

C. Performance of Rayleigh-faded MISO downloading Systems

According to current 3G standard recommendations for cellular (terrestrial) UMTS systems [23 and references therein], [16, Chap.5 and references therein], let us consider now a MISO system where the transmitter utilizes an MRC to build up the signal radiated by the $N_t \geq 1$ transmit antennas, while the user terminal is equipped with a single antenna (e.g., $N_r = 1$). Under the assumption that the (complex) channel coefficients $h_i \in \mathbb{C}$, $i = 1, \dots, N_t$, linking the N_t transmit antennas to the receive one mutually-independent, zero mean, unit-variance Gaussian r.v.s whose values are known (slot-by-slot) to the transmit-MRC, the resulting Shannon' capacity C_{T-MRC} (in *(nats/slot)*) of the overall link from the input of the transmit-MRC to the output of the Maximum-Likelihood (ML) receiver equates [16, Sect.5.4]

$$C_{T-MRC} = \log \left(1 + \frac{\mathcal{E}}{\mathcal{N}_0} \left(\sum_{i=1}^{N_t} |h_i|^2 \right) \right), \quad (\text{nats/slot})$$

where \mathcal{E} is the overall energy radiated by the transmit antennas over a slot-time, while \mathcal{N}_0 (*Joule*) is still the AWGN level measured at the output of the receive antennas. Thus, the channel-state $\sigma \triangleq \frac{1}{\mathcal{N}_0} \left(\sum_{i=1}^{N_t} |h_i|^2 \right)$ is a chi-square central r.v. with $2N_t$ degrees of freedom, so that its pdf equates [16, Sect.5.4]

$$p(\sigma) = \frac{\mathcal{N}_0^{N_t}}{(N_t - 1)!} \sigma^{N_t-1} \exp(-\mathcal{N}_0\sigma), \quad \sigma \geq 0. \quad (48)$$

Hence, since the pdf in (48) may be obtained by replacing N_r by N_t into the pdf (47), we conclude that the performance of a MISO system equipped with a transmit-MRC utilizing N_t antennas is the same as a SIMO system with the same number of receive antennas employing a receive MRC. Therefore, after replacing N_r by N_t , the performance plots of Figs.2, 3, 4 and the above reported remarks about the effectiveness of the optimal policy in (36) still hold unchanged for MRC-based MISO systems.

D. Performance of Rayleigh-faded MIMO Systems equipped with OSTBCs

As final application scenario, we consider a flat-faded MIMO system where the T_X transmitter and R_X receiver are equipped with $N_t \geq 1$ and $N_r \geq 1$ antennas, respectively. After indicating by $h_{ij} \in \mathbb{C}$, $1 \leq i \leq N_t$, $1 \leq j \leq N_r$, the channel-coefficient linking the i -th transmit antenna to the j -th receive one, as

in [16, Sect.6.3] let the coefficients $\{h_{ij}\}$ be zero-mean, unit-variance, mutually-independent Gaussian r.v.s, whose values are known (slot-by-slot) to the receive (but no to the transmitter). Thus, when an OSTBC¹⁶ is employed to convey the data to download, the Shannon' capacity C_{MIMO} (in *nats/slot*) of the overall link going from the input of the Space-Time encoder to the output of the corresponding Spac-Time decoder equates [16, Sect.6.3]

$$C_{MIMO} = r_s \log \left(1 + \frac{\mathcal{E}}{\mathcal{N}_0 N_t} \left(\sum_{i=1}^{N_t} \sum_{j=1}^{N_r} |h_{ij}|^2 \right) \right), \quad (\text{nats/slot}) \quad (49)$$

where $r_s \leq 1$ is the rate of the employed OSTBC, \mathcal{E} (*Joule*) is the energy radiated by the overall N_t transmit antennas over a slot-time, and \mathcal{N}_0 (*Joule*) is the AWGN level measured at the output of each receive antenna. Hence, since the channel-state $\sigma \triangleq \frac{1}{\mathcal{N}_0 N_t} \left(\sum_{i=1}^{N_t} \sum_{j=1}^{N_r} |h_{ij}|^2 \right)$ is now a chi-square central r.v. with $2N_r N_t$ degrees of freedom, its pdf is given by [16, Sect.6.3]

$$p(\sigma) = \frac{1}{(N_t N_r - 1)!} (\mathcal{N}_0 N_t)^{N_t N_r} \sigma^{N_t N_r - 1} \exp(-\mathcal{N}_0 N_t \sigma), \quad \sigma \geq 0. \quad (50)$$

The corresponding average download-time T_0 of the optimal policy (36) is drawn in Fig.5. An examination of the plots in Fig.5 points out that the average download-time T_0 of the optimal policy (36) decreases when N_t and/or N_r increase. However, according to the behavior of the capacity formula in (49), T_0 falls short *faster* by increasing N_r than by increasing N_t . Finally, Fig.6 plots the ratio T_{HE}/T_0 among the average download-times of the heuristic and optimal policies. We may conclude that T_{HE}/T_0 is around 2 for $N_r = 4$ and $N_t = 2$, but it exceed *one order* of magnitude for $N_r = 2$ and N_t limited up to 2.

VI. CONCLUSION AND POSSIBLE GENERALIZATIONS

This paper focuses on the minimization of the average download-time of large-size data over continuous-state wireless channels, when constraints on both total available energy and peak-energy are active. A slotted channel affected by independent fadings over different slots has been considered. By exploiting some general results of the convex Calculus of Variations, at first we developed the optimal energy allocation policy for the general class of concave rate-functions. Thus, we test its performance on three

¹⁶The celebrated Alamouti's code is an example of OSTBC with $N_t = 2$ and $r_s = 1$.

Rayleigh-faded multi-antenna downloading systems and we compared the obtained performance against that of a simple On-Off type heuristic. In a nutshell, the carried out performance comparisons lead to the main conclusion that the performance gain offered by the optimal policy over the considered heuristic one is substantial (e.g., over *one order* of magnitude) in application scenarios characterized by medium-low values of the available average energy per slot (e.g., $K \leq 3.5$ (*Joule/nat*)), and medium/low numbers of transmit/receive antennas (e.g., $N_r \leq 4$ and/or $N_t \leq 4$). The presented results should be no considered conclusive but, on the contrary, they represent, indeed, the tip of an iceberg. Just a first hint for future investigations, the generalization of the carried-out analysis to the case of multi-channel systems with multidimensional continuous channel-states could be accomplished and, then, applied for the optimized design of multiuser (possibly, multiantenna) wireless broadcast systems. As a second hint, we are aware that the model adopted for the downlink channel may no cover all the features of actual wireless data-links. Although we choosed it because it captures several key-aspects of actual links and may be rigorously analyzed, nevertheless it could be of interest to generalize the carried out analysis when both the assumptions of perfect CSI at the transmitter and independent fading over different slots are relaxed.

To date, those mentioned generalizations do no look, indeed, direct, and currently they are under investigation by the authors.

APPENDIX I — PROOF OF THE PROPOSITION 1

Since in our model the channel-state is continuous-valued, the approach pursued in [15, Chap.3] (and based on the Dynamic Programming tool) falls short. So, for proving the validity of the *Proposition 1*, we resort to exploit some basic properties of the convex functional analysis, as detailed in the sequel of this Appendix. To begin with, let us consider an arbitrary energy-allocation policy $\varepsilon(.,.;.) \in \Omega$ *feasible* for the problem in (10), and let T be the average-time (in multiple of the slot-time) requested by $\varepsilon(.,.;.)$ to download the assigned *IUs*. Furthermore, let $\hat{\varepsilon}(\sigma) \triangleq E\{\varepsilon(.,.;.)|\sigma\}$ (*Joule*) and $\hat{\Delta}(\sigma) \triangleq E\{IU(t)|\sigma\}$ be the corresponding average energy per slot radiated when the channel is in the σ -state, and the resulting average number of conveyed *IUs*, respectively. Finally, let $d\hat{\tau}(\sigma) \leq 1$ be the *fraction* of the time (between

times $t = 1$ and $t = T$) that the channel-state falls in the interval $[\sigma, \sigma + d\sigma)$, so that

$$d\hat{t}(\sigma) \triangleq T d\hat{\tau}(\sigma), \quad (\text{slot}) \quad (\text{A.1})$$

is the total time spent by the channel-state in the interval $[\sigma, \sigma + d\sigma)$ during the overall download. Being the rate-function in (1) concave for $\mathcal{E} \geq 0$ (see eq.(3)), a direct application of the Jensen's inequality [17,p.77] leads to the following upper-bound on $\hat{\Delta}(\sigma)$:

$$\hat{\Delta}(\sigma) \triangleq E \{IU(t)|\sigma\} = E \{\mathcal{R}(\mathcal{E}; \sigma)|\sigma\} \leq \mathcal{R}(E \{\mathcal{E}|\sigma\}; \sigma) \triangleq \mathcal{R}(\hat{\varepsilon}(\sigma); \sigma). \quad (\text{A.2})$$

Furthermore, an exploitation of (A.1) allows us to express the available total energy \mathcal{E}_{tot} in (5) and the overall number Δ of IUs to be downloaded as in

$$\mathcal{E}_{tot} \equiv \int_{\sigma=0}^{\infty} \hat{\varepsilon}(\sigma) d\hat{t}(\sigma) = T \int_{\sigma=0}^{\infty} \hat{\varepsilon}(\sigma) d\hat{\tau}(\sigma), \quad (\text{A.3})$$

$$\Delta \equiv \int_{\sigma=0}^{\infty} \hat{\Delta}(\sigma) d\hat{t}(\sigma) = T \int_{\sigma=0}^{\infty} \hat{\Delta}(\sigma) d\hat{\tau}(\sigma), \quad (\text{A.4})$$

so that the constraint in (10.1) on the total available energy may be equivalently rewritten as in

$$\int_{\sigma=0}^{\infty} \hat{\varepsilon}(\sigma) d\hat{\tau}(\sigma) \leq \int_{\sigma=0}^{\infty} \hat{\Delta}(\sigma) d\hat{\tau}(\sigma). \quad (\text{A.5})$$

Now, since the random sequence $\{\sigma(t) \in \mathbb{R}_0^+, t \geq 1\}$ of the channel-states has been assumed i.i.d. (and, then it is ergodic), an application of the strong law of the large numbers guarantees that, for large Δ , the above introduced time-fraction $d\hat{\tau}(\sigma)$ converges to the corresponding probability. Thus, for large Δ we can write

$$d\hat{\tau}(\sigma) = P(\sigma \leq \sigma < \sigma + d\sigma) \equiv p(\sigma) d\sigma, \quad (\text{A.6})$$

that, in turn, leads to (see eq.(A.1))

$$d\hat{t}(\sigma) \equiv T d\hat{\tau}(\sigma) = T p(\sigma) d\sigma. \quad (\text{A.7})$$

As a consequence, after inserting (A.2) and (A.7) into (A.5), for large Δ the inequality (A.5) gives arise to the following constraint on the above defined conditional average energy $\hat{\varepsilon}(\sigma)$:

$$\int_0^{\infty} \hat{\varepsilon}(\sigma) p(\sigma) d\sigma \leq K \int_0^{\infty} \mathcal{R}(\hat{\varepsilon}(\sigma); \sigma) p(\sigma) d\sigma. \quad (\text{A.8})$$

Eq.(A.8) proves that, for large Δ , the conditional average energy $\widehat{\varepsilon}(\sigma)$ associated to any energy-allocation policy $\varepsilon(.,.;.) \in \Omega$ feasible for the problem in (10) *satisfies* the constraint in (12.1). Now, since $\widehat{\varepsilon}(\sigma)$ also meets the limit (12.2) on the allowed peak-energy¹⁷, we may claim that, for large Δ , the conditional average energy $\widehat{\varepsilon}(\sigma)$ satisfies *both* constraints (12.1), (12.2). Since, by definition, $\widehat{\varepsilon}(\sigma)$ only depends on the channel-state, we conclude that, for large Δ , the average conditional energy $\widehat{\varepsilon}(\sigma)$ associated to any policy $\varepsilon(.,.;.) \in \Omega$ feasible for the problem in (10) *falls* in the set \mathcal{A} in (11) and, in addition, it is *feasible* for the problem in (12), e.g.,

$$\text{if } \varepsilon(.,.;.) \in \Omega \text{ is feasible for (10)} \rightarrow \widehat{\varepsilon}(\sigma) \in \mathcal{A} \text{ is feasible for (12)}. \quad (\text{A.9})$$

Now, after indicating by r_0 the rate in (13) attained by the optimal policy $\varepsilon_0(.) \in \mathcal{A}$ solution of the constrained problem (12), let r be the average rate conveyed by the policy $\varepsilon(.,.;.) \in \Omega$ feasible for the problem (10). Since the corresponding $\widehat{\varepsilon}(\sigma)$ is feasible for (12) (see eq.(A.9)), an exploitation of the inequality (A.2) allows us to write

$$r \triangleq \text{E} \left\{ \widehat{\Delta}(\sigma) \right\} \stackrel{(a)}{\leq} \text{E} \left\{ \mathcal{R}(\widehat{\varepsilon}(\sigma); \sigma) \right\} \stackrel{(b)}{\leq} r_0, \quad (\text{A.10})$$

where (a) follows from (A.2), while (b) stems from the feasibility property of $\widehat{\varepsilon}(\sigma)$ in (A.9). Overall, since (A.10) proves that, for large Δ , the rate r conveyed by any energy-allocation policy $\varepsilon(.,.;.) \in \Omega$ feasible for the problem (10) *cannot exceed* the rate r_0 of the optimal policy $\varepsilon_0(\sigma) \in \mathcal{A}$ solution of the problem (12), the proof of the *Proposition1* is completed. \blacksquare

APPENDIX II — PROOF OF THE PROPOSITION 3

Since the energy allocation policy in (21) meets both constraints (12.1), (12.2), it is feasible. In particular, it is the *unique* feasible policy when $K = 0$ (see eq.(12.1)). Thus, our task is to prove that

¹⁷In fact, by definition, any policy $\varepsilon(.,.;.) \in \Omega$ feasible for the problem in (10) satisfies the constraint (10.2), so that we can write

$$0 \leq \varepsilon \left(\sigma; \mathcal{E}^{(r)}; \Delta^{(r)} \right) \leq \mathcal{E}_{max}.$$

Thus, by performing the conditional expectation $\text{E}\{.\mid\sigma\}$ of the terms involved by the above inequality, we directly arrive at

$$0 \leq \widehat{\varepsilon}(\sigma) \leq \mathcal{E}_{max}.$$

the allocation policy (21) is *still the unique* feasible policy for any $K \in [0, K_L]$. Towards this end, we begin to observe that the rate-function satisfies the following chain of inequalities:

$$\mathcal{R}(\mathcal{E}; \sigma) \stackrel{(a)}{\leq} \mathcal{R}(0; \sigma) + m(\sigma)\varepsilon(\sigma) \stackrel{(b)}{=} m(\sigma)\varepsilon(\sigma), \quad (\text{A.11})$$

where (a) arises from the concavity of the rate-function [17,p.70], while (b) stems from the assumption in (2). Thus, after inserting (A.11) into the constraint (12.1), we conclude that any energy allocation policy $\varepsilon(\sigma)$ satisfying (12.1) must *also meet* the following inequality:

$$\int_0^\infty \varepsilon(\sigma)p(\sigma)d\sigma \leq K \int_0^\infty m(\sigma)\varepsilon(\sigma)p(\sigma)d\sigma.$$

Since this last may be re-cast as in

$$\int_0^\infty [1 - Km(\sigma)] \varepsilon(\sigma)p(\sigma)d\sigma \leq 0, \quad (\text{A.12})$$

from (A.12) we deduce that, when the following condition is met:

$$Km(\sigma) \leq 1, \text{ for any } \sigma \geq 0, \quad (\text{A.13})$$

thus the *unique* feasible energy-allocation policy satisfying (A.12) is that reported in (21). Finally, after noting that the definitions of K_L in (20) guarantees that (A.13) is satisfied for any $K \leq K_L$, the proof of the *Proposition 3* is completed. ■

APPENDIX III — SLATER'S CONDITIONS AND STRONG DUALITY

Being the primal-problem (12) convex, Slater's condition guarantees that the (constrained) minimum in (22) attains the (constrained) maximum in (12). Thus, strong duality holds and, for $K \in]K_L, K_H[$, the solution of the primal-problem (12) may be obtained by solving the dual-problem (22). Specifically, since the point-wise upper and lower constraints in (12.2) are affine, testing Slater's condition requires that, for $K \in]K_L, K_H[$, there exists at least one non-negative energy-allocation policy $\varepsilon(\sigma) \in \mathcal{A}$ limited up to \mathcal{E}_{max} that meets the integral constraint (12.1) with *strict* inequality [17, Sect.5.2]. Being K_L in (20) the largest K value such that $Km(\sigma) \leq 1$ for $\sigma \geq 0$, we deduce that, for $K > K_L$, there exists at least one state-value $\bar{\sigma}$ such that $Km(\bar{\sigma}) > 1$. Therefore, due to the continuity property of the derivative

function $m(\sigma)$ in (19) (see Sect.II-A), there exists a neighborhood \mathcal{I} of $\bar{\sigma}$ such that $Km(\sigma) > 1$ for any $\sigma \in \mathcal{I}$. Hence, after indicating by

$$\mathcal{B} \triangleq \{\varepsilon(\sigma) \in \mathcal{A} : 0 \leq \varepsilon(\sigma) \leq \mathcal{E}_{max} \text{ for } \sigma \in \mathcal{I}, \text{ and } \varepsilon(\sigma) = 0 \text{ for } \sigma \notin \mathcal{I}\}, \quad (\text{A.14})$$

the sub-set of the energy-allocation policies meeting the point-wise constraints (12.2) and vanishing out the set \mathcal{I} , from the outset we directly deduce that the following *strict* inequality must take place:

$$\int \varepsilon(\sigma)p(\sigma)d\sigma < \int Km(\sigma)\varepsilon(\sigma)p(\sigma)d\sigma, \text{ for any } \varepsilon(\cdot) \in \mathcal{B}. \quad (\text{A.15})$$

Thereinafter, the test of Slater's condition proceeds by contradiction. Specifically, let us assume that there does not exist any energy-allocation policy $\varepsilon(\cdot) \in \mathcal{A}$ meeting the constraint (12.1) with strict inequality.

In this case, we have

$$\int \varepsilon(\sigma)p(\sigma)d\sigma \geq K \int \mathcal{R}(\varepsilon(\sigma); \sigma)p(\sigma)d\sigma, \text{ for any } \varepsilon(\cdot) \in \mathcal{B}. \quad (\text{A.16})$$

Let us consider now the On-Off policy $\bar{\varepsilon}(\cdot) \in \mathcal{B}$ so defined:

$$\bar{\varepsilon}(\sigma) \triangleq \begin{cases} \lambda, & \text{for } \sigma \in \mathcal{I}, \\ 0, & \text{for } \sigma \notin \mathcal{I}, \end{cases} \quad (\text{A.17})$$

with λ assigned and falling in the (closed) interval $[0, \mathcal{E}_{max}]$. Thus, for the policy (A.17), (A.15) and (A.16) become, respectively

$$P_0 < K \int_{\sigma \in \mathcal{I}} m(\sigma)p(\sigma)d\sigma, \quad (\text{A.18})$$

and

$$\lambda P_0 \geq K \int_{\sigma \in \mathcal{I}} \mathcal{R}(\lambda; \sigma)p(\sigma)d\sigma, \quad (\text{A.19})$$

where $P_0 \triangleq P(\sigma \in \mathcal{I})$ is the probability that the channel-state σ falls in the set \mathcal{I} . Since (A.19) holds for any $\lambda \in [0, \mathcal{E}_{max}]$, it must also hold for vanishing λ . Thus, being $\mathcal{R}(\lambda; \sigma) = \lambda m(\sigma)$ for small λ (see eq.(A.11)), by passing to the limit for $\lambda \rightarrow 0$ eq.(A.19), we obtain that the following constraint must also be met:

$$P_0 \geq K \int_{\sigma \in \mathcal{I}} m(\sigma)p(\sigma)d\sigma. \quad (\text{A.20})$$

Being the conditions (A.18), (A.19) in contrast, we conclude that there must exist at least one energy-allocation policy $\varepsilon(\cdot) \in \mathcal{B}$ satisfying the integral constraint (12.1) with *strict* inequality. Hence, the test of Slater's conditions is now completed. ■

APPENDIX IV — PROOF OF THE PROPOSITION 4

Thus, Slater's condition guarantees that the optimal energy allocation policy $\varepsilon_0(\cdot; \cdot)$ solution of the (primal) problem (12) may be computed by solving the corresponding dual-problem (22). Towards this end, in the following sub-Appendix IV.A we prove that the optimal policy retains the form reported in (28), while the last sub-Appendix IV.B is devoted to derive the expressions (26), (27), (30) for the parameters involved by the optimal policy.

IV.A — Form of the Optimal Policy

The first step is to compute the dual-function $g(\mu)$ in (23). For this purpose, we begin to note that, for any fixed $\mu \geq 0$, this computation is equivalent to solve the following constrained maximization problem:

$$\max_{\varepsilon(\sigma; \mu)} J(\varepsilon(\sigma; \mu)), \quad (\text{A.21})$$

$$s.t. : 0 \leq \varepsilon(\sigma; \mu) \leq \mathcal{E}_{max}, \quad (\text{A.21.1})$$

with the J -functional defined as in (see eq.(23))

$$J(\varepsilon(\sigma; \mu)) \triangleq \int_0^\infty F(\sigma; \varepsilon(\sigma; \mu); \mu) d\sigma. \quad (\text{A.22})$$

Due to the (assumed) concavity property in (3) of the rate-function, both $F(\cdot; \cdot; \cdot)$ in (24) and $J(\cdot)$ in (A.22) are concave over $\varepsilon(\cdot; \cdot) \geq 0$, so that the constrained maximization in (A.21) is an instance of concave Calculus of Variations [18]. Thus, after indicating by $\varepsilon_0(\sigma; \mu)$ the policy achieving the constrained maximum of (A.21), the concave nature of this problem leads to the following basic results on the form assumed by $\varepsilon_0(\cdot; \cdot)$.

Lemma 1: (Form assumed by the Optimal Policy)

Under the assumptions on the rate-function previously reported in Sect.II-A, for any K falling in the interval $]K_L, K_H[$ we have that:

- i) for any assigned $\mu \geq 0$, there exist two *unique* thresholds $\sigma_L(\mu)$ and $\sigma_H(\mu)$ on the channel-state meeting the ordering relationship:

$$0 \leq \sigma_L(\mu) \leq \sigma_H(\mu) \leq +\infty, \quad (\text{A.23})$$

that allow us to compute the optimal policy $\varepsilon_0(.;.)$ as in

$$\varepsilon_0(\sigma; \mu) = \begin{cases} 0, & \text{for } \sigma \in [0, \sigma_L(\mu)], \\ \max \{0; \min \{\varepsilon^*(\sigma; \mu); \mathcal{E}_{max}\}\}, & \text{for } \sigma \in]\sigma_L(\mu), \sigma_H(\mu)[, \\ \mathcal{E}_{max}, & \text{for } \sigma \in [\sigma_H(\mu), \infty[, \end{cases} \quad (\text{A.24})$$

where $\varepsilon^*(\sigma; \mu)$ is the *unique* solution of the functional equation (25) over the (open) interval $\sigma \in]\sigma_L(\mu), \sigma_H(\mu)[$;

- ii) for any assigned $\mu \geq 0$, the optimal policy $\varepsilon_0(\sigma; \mu)$ in (A.24) is *no decreasing* in the σ -variable;
 iii) finally, when $\sigma_L(\mu) < \sigma_H(\mu)$ (e.g., $\sigma_L(\mu) \neq \sigma_H(\mu)$), then the optimal policy $\varepsilon_0(\sigma; \mu)$ in (A.24) is also *continuous* for $\sigma \geq 0$.

Proof.

- i) After fixing $\mu \geq 0$, let us consider the following *unconstrained* maximization problem:

$$\max_{\varepsilon(\sigma; \mu)} J(\varepsilon(\sigma; \mu)). \quad (\text{A.25})$$

Since $F(.,.;.)$ in (24) is concave in the \mathcal{E} -variable, a policy $\varepsilon^*(\sigma; \mu)$ attains the (unconstrained) maximum in (A.25) *if and only if* it satisfies the functional equation (25). However, in general, the solution $\varepsilon^*(\sigma; \mu)$ of (25) is no guaranteed to exist for *any* $\sigma \geq 0$. Thus, at first, let $\sigma_1 \geq 0$ be a channel-state such that

$$F_\varepsilon(\sigma_1; \varepsilon(\sigma_1; \mu); \mu) < 0, \quad \text{for any } \varepsilon(\sigma_1; \mu) \geq 0. \quad (\text{A.26})$$

Since the rate-function has been assumed no decreasing over $\sigma \geq 0$ (see Sect.II-A), thus $F_\varepsilon(.,.;.)$ is negative over the overall interval $\sigma \in [0, \sigma_1]$, so that the functional $J(.)$ in (A.22) *does no decrease* for vanishing $\varepsilon(.,.)$. As a consequence, the following policy: $\varepsilon^*(\sigma; \mu) \equiv 0$ for any $\sigma \in [0, \sigma_1]$, attains the maximum in the constrained problem (A.21), and it coincides with the optimal one $\varepsilon_0(.,.)$, so that we can write

$$\varepsilon_0(\sigma; \mu) = 0, \quad \text{for any } \sigma \leq \sigma_1. \quad (\text{A.27})$$

Now, let us indicate by σ_3 a channel state such that

$$F_\varepsilon(\sigma_3; \varepsilon(\sigma_3; \mu); \mu) > 0, \quad \text{for any } \varepsilon(\sigma_3; \mu) \geq 0.$$

In this case, F_ε must be strictly positive over the overall interval $\sigma \in [\sigma_3, +\infty[$, so that the J -functional in (A.22) is *no decreasing* for increasing $\varepsilon(\cdot; \cdot)$. As consequence, the following policy: $\varepsilon^*(\sigma; \mu) \equiv \mathcal{E}_{max}$ for any $\sigma \in [\sigma_3, +\infty[$, attains the maximum in (A.21), and we conclude that

$$\varepsilon_0(\sigma; \mu) = \mathcal{E}_{max}, \quad \text{for any } \sigma \geq \sigma_3, \quad (\text{A.28})$$

is the optimal policy for the constrained maximization problem (A.21). Therefore, from the outset it stems that the thresholds $\sigma_L(\mu), \sigma_H(\mu)$ defined as in

$$\sigma_L(\mu) \text{ is the biggest } \sigma \geq 0 \text{ such that: } F_\varepsilon(\sigma; \varepsilon(\sigma; \mu); \mu) < 0, \text{ for any } \varepsilon(\cdot; \cdot) \geq 0, \quad (\text{A.29})$$

and

$$\sigma_H(\mu) \text{ is the smallest } \sigma \geq 0 \text{ such that: } F_\varepsilon(\sigma; \varepsilon(\sigma; \mu); \mu) > 0, \text{ for any } \varepsilon(\cdot; \cdot) \geq 0, \quad (\text{A.30})$$

must satisfy the ordering relationship in (A.23). In addition, the optimal policy $\varepsilon_0(\sigma; \mu)$ *vanishes* over $\sigma \in [0, \sigma_L(\mu)]$, while it *attains* \mathcal{E}_{max} for $\sigma \geq \sigma_H(\mu)$ (see eqs. (A.27), (A.28)).

As last case, let σ an arbitrary value of the channel-state falling in the *open* interval $]\sigma_L(\mu), \sigma_H(\mu)[$. Due to the concavity of the rate-function (see eq.(3)), the derivative $F_\varepsilon(\cdot; \cdot; \cdot)$ is (strictly) decreasing for $\mathcal{E} \geq 0$. So, from (3) and (4) it follows that the solution $\varepsilon^*(\sigma, \mu)$ of eq.(25) *exists*, and it is *unique* and *nondecreasing* over the overall interval $\sigma \in]\sigma_L(\mu), \sigma_H(\mu)[$. However, in this case is no longer guaranteed that $\varepsilon^*(\sigma; \mu)$ meets the constraints in (A.21.1) for any σ falling in the open interval $]\sigma_L(\mu), \sigma_H(\mu)[$. Since the Karush-Kuhn-Tucker (KKT) conditions [17, Chap.5] for the concave optimization problem in (A.21) assume the form reported in the sequel ¹⁸:

$$F_\varepsilon(\sigma; \varepsilon(\sigma; \mu); \mu) + [\lambda_2(\sigma) - \lambda_1(\sigma)] = 0, \quad (\text{A.31})$$

¹⁸ $\lambda_1(\sigma)$ and $\lambda_2(\sigma)$ are the (nonnegative) Lagrange multipliers associated to the box-constraints in (A.21.1).

$$\lambda_1(\sigma) [\varepsilon(\sigma; \mu) - \mathcal{E}_{max}] = 0, \quad (\text{A.32})$$

$$\lambda_2(\sigma) \varepsilon(\sigma; \mu) = 0, \quad (\text{A.33})$$

thus, by inspection it can be tested that the following function-tuple:

$$\tilde{\varepsilon}(\sigma; \mu) \triangleq \max \{0; \min \{\varepsilon^*(\sigma; \mu); \mathcal{E}_{max}\}\}, \quad (\text{A.34})$$

$$\tilde{\lambda}_1(\sigma) \triangleq \begin{cases} F_\varepsilon(\sigma; \mathcal{E}_{max}; \mu), & \text{for any } \sigma : \varepsilon^*(\sigma; \mu) \geq \mathcal{E}_{max}, \\ 0, & \text{otherwise,} \end{cases} \quad (\text{A.35})$$

and

$$\tilde{\lambda}_2(\sigma) \triangleq \begin{cases} -F_\varepsilon(\sigma; 0; \mu), & \text{for any } \sigma : \varepsilon^*(\sigma; \mu) \leq 0, \\ 0, & \text{otherwise,} \end{cases} \quad (\text{A.36})$$

satisfies the above listed KKT conditions over the overall interval $\sigma \in]\sigma_L(\mu), \sigma_H(\mu)[$. As a consequence, the expression in (A.34) gives the form assumed by the optimal policy $\varepsilon_0(\sigma; \mu)$ over the interval $\sigma \in]\sigma_L(\mu), \sigma_H(\mu)[$. This concludes the proof of the *i*)-part of *Lemma 1*.

ii) We have already tested in the above first part of this proof that, for any assigned μ , the solution $\varepsilon^*(\sigma; \mu)$ of the functional equation (25) *does not decrease* for increasing σ . Since $\varepsilon_0(\sigma; \mu)$ assumes the form in (A.24), this suffices to guarantee that also $\varepsilon_0(\cdot; \cdot)$ *does not decrease* for increasing σ -values.

iii) When $\sigma_L(\mu)$ equates $\sigma_H(\mu)$, the optimal policy $\varepsilon_0(\cdot; \cdot)$ in (A.24) reduces to an On-Off one, so that $\varepsilon_0(\sigma; \mu)$ is continuous for any $\sigma \neq \sigma_L(\mu) = \sigma_H(\mu)$.

Thus, let us consider the case $\sigma_L(\mu) < \sigma_H(\mu)$. In this case, the continuity property of the rate-function and its derivatives (see Sect.II-A) guarantees that Dini's theorem holds, so that the solution $\varepsilon^*(\sigma; \mu)$ of the equation (25) is continuous (and also derivable) over the interval $\sigma \in]\sigma_L(\mu), \sigma_H(\mu)[$. As a direct consequence, $\varepsilon_0(\sigma; \mu)$ in (A.24) is continuous at least for any $\sigma \geq 0$ different from $\sigma_L(\mu)$ and $\sigma_H(\mu)$. Therefore, for completing the proof, it remains only to prove that the optimal policy $\varepsilon_0(\cdot; \cdot)$ in (A.24) is continuous also at $\sigma = \sigma_L(\mu)$ and $\sigma = \sigma_H(\mu)$.

This proof may be carried out by contradiction. Specifically, let us assume

$$\lim_{\sigma \rightarrow \sigma_L(\mu)^+} \varepsilon^*(\sigma; \mu) = \mathcal{E}_1 > 0, \quad (\text{A.37})$$

so that $\varepsilon_0(\sigma; \mu)$ in (A.24) is not continuous at $\sigma = \sigma_L(\mu)$. Thus, since $F_\varepsilon(\cdot; \cdot; \cdot)$ is continuous and strictly decreasing for increasing \mathcal{E} values, (A.37) implies that there must exist (at least) two perturbing values $\delta_1 \in]0, \mathcal{E}_1[$ and $\delta_2 \in]0, \sigma_L(\mu)[$ such that

$$F_\varepsilon(\sigma_L(\mu) - \delta_2; \mathcal{E}_1 - \delta_1; \mu) > 0. \quad (\text{A.38})$$

However, (A.38) is not feasible, because the point: $\sigma_L(\mu) - \delta_2$ falls in the interval $]0, \sigma_L(\mu)[$, so that $F_\varepsilon(\sigma_L(\mu) - \delta_2; \mathcal{E}_1 - \delta_1; \mu)$ must be strictly negative. Hence, the optimal policy $\varepsilon_0(\cdot; \cdot)$ in (A.24) must be continuous at $\sigma = \sigma_L(\mu)$.

By proceeding in a dual-way, it can be proved that $\varepsilon_0(\cdot; \cdot)$ is also continuous at $\sigma = \sigma_H(\mu)$.

The proof of *Lemma 1* is now completed. ■

IV.B — Evaluation of the parameters involved by the optimal policy

The \mathcal{E} -derivative of the F -function in (24) equates

$$F_\varepsilon(\sigma; \mathcal{E}; \mu) = p(\sigma) [(1 + K\mu)\mathcal{R}_\varepsilon(\mathcal{E}; \sigma) - \mu]. \quad (\text{A.39})$$

Thus, by introducing (A.39) into the defining relationships (A.29), (A.30), we have that these last may be put in the equivalent forms reported by eqs. (26), (27), respectively. In addition, for σ falling in the interval $] \sigma_L(\mu), \sigma_H(\mu)[$, eq. (25) becomes

$$\mathcal{R}_\varepsilon(\mathcal{E}; \sigma) = \frac{\mu}{1 + K\mu}, \quad (\text{A.40})$$

that, in turn, leads to the (closed-form) expression (30) for $\varepsilon^*(\sigma; \mu)$.

About the evaluation of the optimal value μ^* of the dual-variable, we note that the dual-function $g(\mu)$ in (23) is convex over $\mu \geq 0$. Since, by definition, $g(\mu)$ equates (see eq.(23))

$$g(\mu) = \int_0^\infty F(\sigma; \varepsilon_0(\sigma; \mu); \mu) d\sigma, \quad (\text{A.41})$$

thus, the optimal value μ^* of the dual-variable may be directly computed by solving (with respect μ) the following algebraic equation:

$$\frac{dg(\mu)}{d\mu} \equiv \int_0^\infty F_\mu(\sigma; \varepsilon_0(\sigma; \mu); \mu) d\sigma = 0. \quad (\text{A.42})$$

Overall, after noting that $\varepsilon_0(\sigma; \mu)$ and, then, $F(\sigma; \varepsilon_0(\sigma; \mu); \mu)$ both vanish for any $\sigma \leq \sigma_L(\mu)$ (see eqs. (24), (28)), from (A.42) we directly arrive at the final relationship in (29). ■

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SYSTEM PARAMETER	MEANING/ROLE
$\Delta(IU)$	Number of IUs to be downloaded
\mathcal{E}_{tot} (Joule)	Overall Energy available for the download
$\mathcal{E}(t)$ (Joule)	Energy radiated at slot t
\mathcal{E}_{max} (Joule)	Peak-Energy per slot
$\mathcal{E}^{(r)}(t)$ (Joule)	Residual Energy available at the beginning of slot t
$\Delta^{(r)}(t)$ (IU)	Residual numbers of IUs to be still downloaded at the beginning of slot t
$K \triangleq \mathcal{E}_{tot}/\Delta$ (Joule/ IU)	Available average energy per IU
T (slot)	Time (in multiple of the slot-time) required for the download
$r \triangleq E\{IU(t)\} \equiv \overline{IU}$ (IU /slot)	Average number of downloaded IUs per slot

TABLE I

LIST OF THE MAIN SYSTEM PARAMETERS INVOLVED BY THE AFFORDED OPTIMIZATION PROBLEM

1. Compute the threshold K_H and K_L via the eqs.(17), (20).
2. If $K \leq K_L$, then:
 - 2.1 Set $\varepsilon_0(\sigma) = 0$, a.s. for $\sigma \geq 0$;
 - 2.2 Set $r_0 = 0$;
 - 2.3 Set $T_0 = \infty$;
 - 2.4 Stop.
3. If $K \geq K_H$, then:
 - 3.1 Set $\varepsilon_0(\sigma) = \mathcal{E}_{max}$, for $\sigma \geq 0$;
 - 3.2 Compute r_{max} via eq. (16);
 - 3.3 Set $r_0 = r_{max}$;
 - 3.4 Set $T_0 = \Delta/r_{max}$;
 - 3.5 Stop.
4. If $K \in]K_L, K_H[$, then:
 - 4.1 Compute $\sigma_L(\mu)$ and $\sigma_H(\mu)$ via eqs.(26), (27);
 - 4.2 Compute μ^* by solving eq.(29);
 - 4.3 Compute $\varepsilon_0(\sigma) \equiv \varepsilon_0(\sigma; \mu^*)$ via eq.(28);
 - 4.4 Compute r_0, T_0 via eqs. (13), (14);
 - 4.5 Stop.

TABLE II

A PSEUDO-PROGRAM TO COMPUTE THE OVERALL OPTIMAL ENERGY ALLOCATION POLICY $\varepsilon_0(\sigma)$

PARAMETERS	ASSUMED EXPRESSION
K_L	0
K_H	$\mathcal{E}_{max} / (\int_0^\infty \log(1 + \mathcal{E}_{max}\sigma) p(\sigma) d\sigma)$
$\sigma_L(\mu^*)$	$(K + 1/\mu^*)^{-1}$
$\sigma_H(\mu^*)$	$(K - \mathcal{E}_{max} + 1/\mu^*)^{-1}$, for $(K - \mathcal{E}_{max} + 1/\mu^*) \geq 0$ ∞ , otherwise
$P(\sigma_L(\mu) \leq \sigma \leq \sigma_H(\mu))$	$\int_{\sigma_L(\mu)}^{\sigma_H(\mu)} p(\sigma) d\sigma$
$E\{K \log \sigma + \frac{1}{\sigma} \sigma_L(\mu) \leq \sigma \leq \sigma_H(\mu)\}$	$\int_{\sigma_L(\mu)}^{\sigma_H(\mu)} (K \log \sigma + \frac{1}{\sigma}) p(\sigma) d\sigma$
$P(\sigma \geq \sigma_H(\mu))$	$\int_{\sigma_H(\mu)}^\infty p(\sigma) d\sigma$
$E\{\log(1 + \sigma \mathcal{E}_{max}) \sigma \geq \sigma_H(\mu)\}$	$\int_{\sigma_H(\mu)}^\infty \log(1 + \mathcal{E}_{max}\sigma) p(\sigma) d\sigma$

TABLE III

EXPRESSION OF THE PARAMETERS CHARACTERIZING THE OPTIMAL POLICY FOR THE LOGARITHMIC RATE-FUNCTION

IN (35)

PARAMETERS	ASSUMED EXPRESSION
K_L	0
K_H	$(\mathcal{E}_{max})^{1-\alpha} / (\int_0^\infty \sigma^\alpha p(\sigma) d\sigma)$
$\sigma_L(\mu^*)$	0
$\sigma_H(\mu^*)$	$\left\{ (\mathcal{E}_{max})^{\frac{1-\alpha}{\alpha}} / [\alpha (K + 1/\mu^*)] \right\}^{1/\alpha}$
$E\{\sigma^{\alpha/1-\alpha} \sigma \leq \sigma_H(\mu)\}$	$\int_0^{\sigma_H(\mu)} \sigma^{\frac{\alpha}{1-\alpha}} p(\sigma) d\sigma$
$P(\sigma \geq \sigma_H(\mu))$	$\int_{\sigma_H(\mu)}^\infty p(\sigma) d\sigma$
$E\{\sigma^\alpha \sigma \geq \sigma_H(\mu)\}$	$\int_{\sigma_H(\mu)}^\infty \sigma^\alpha p(\sigma) d\sigma$

TABLE IV

EXPRESSION OF THE PARAMETERS CHARACTERIZING THE OPTIMAL POLICY FOR THE α -POWERED RATE-FUNCTION IN (38)

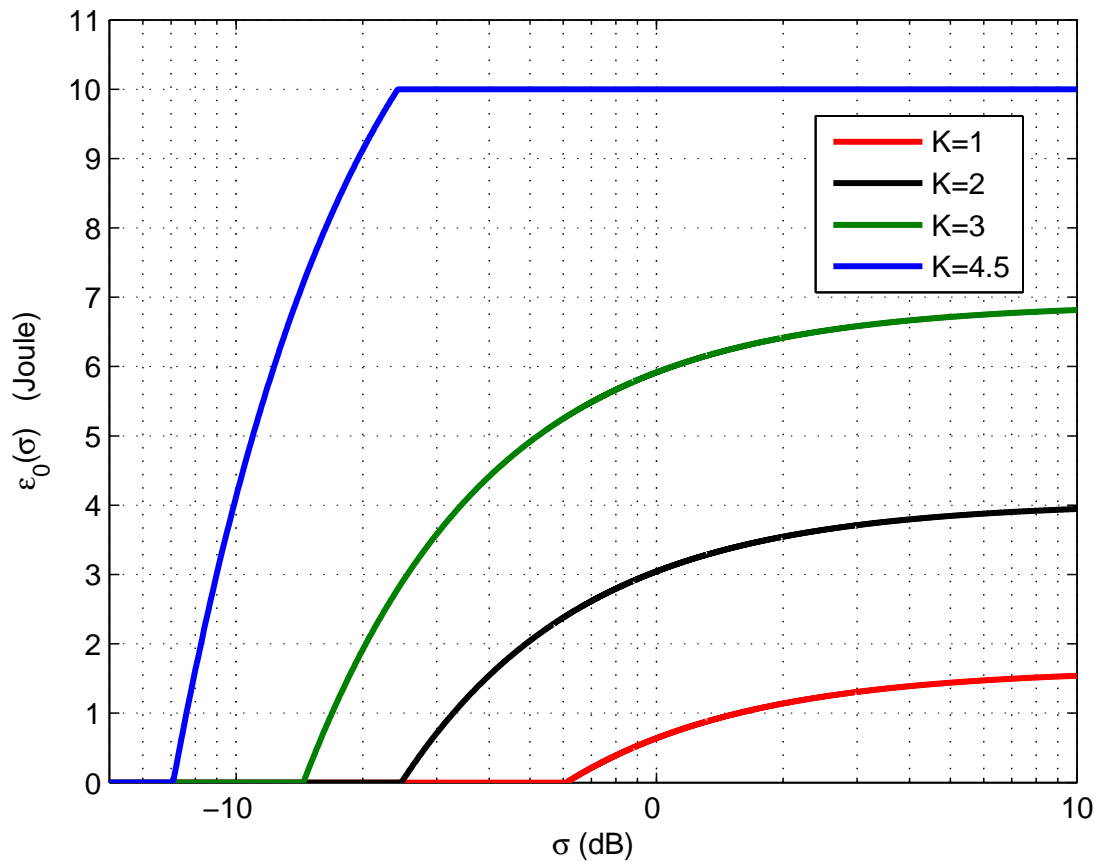


Fig. 1. Behavior of the optimal policy (36) for a SISO (e.g., $N_r = N_t = 1$) system with $\mathcal{N}_0 = 1(\text{Joule})$, $\mathcal{E}_{max} = 10(\text{Joule})$, and K ranging from $1(\text{Joule/slot})$ to $4.5(\text{Joule/slot})$.

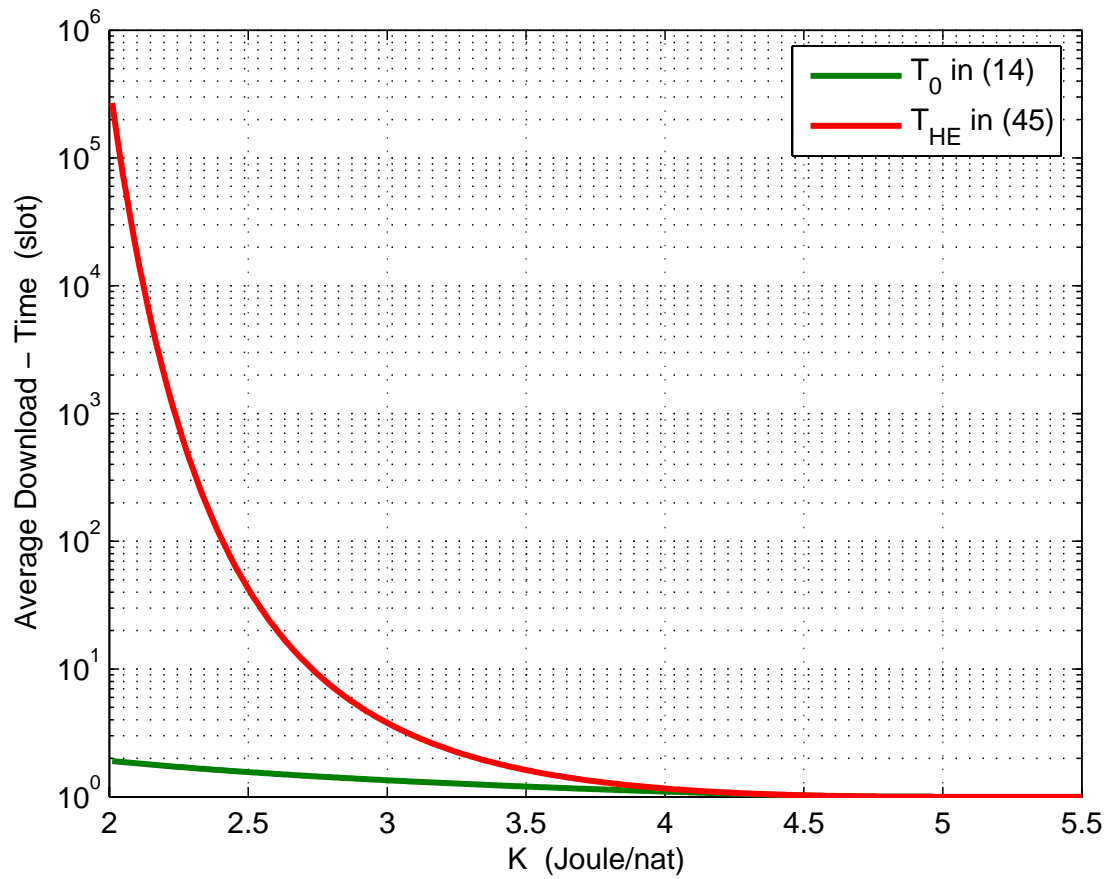


Fig. 2. Download-times versus K -values for a SISO (e.g., $N_r = N_t = 1$) system with $\mathcal{N}_0 = 1(\text{Joule})$, $\mathcal{E}_{max} = 10(\text{Joule})$, and $\Delta = 100(\text{Mnat})$.

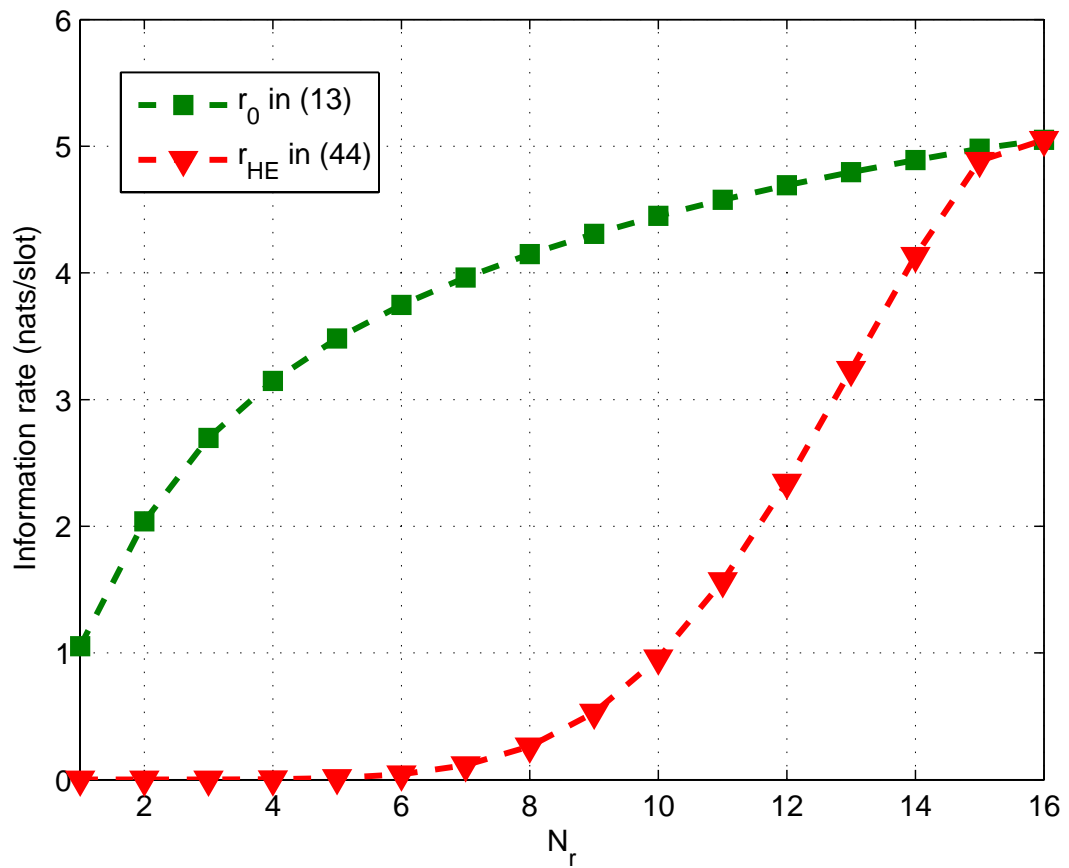


Fig. 3. Average download-rates for the optimal policy and the heuristic one versus the number N_r of receive antennas for a SIMO-MRC system with $\mathcal{N}_0 = 1(\text{Joule})$, $\mathcal{E}_{max} = 10(\text{Joule})$, and $K = 2(\text{Joule/nat})$.

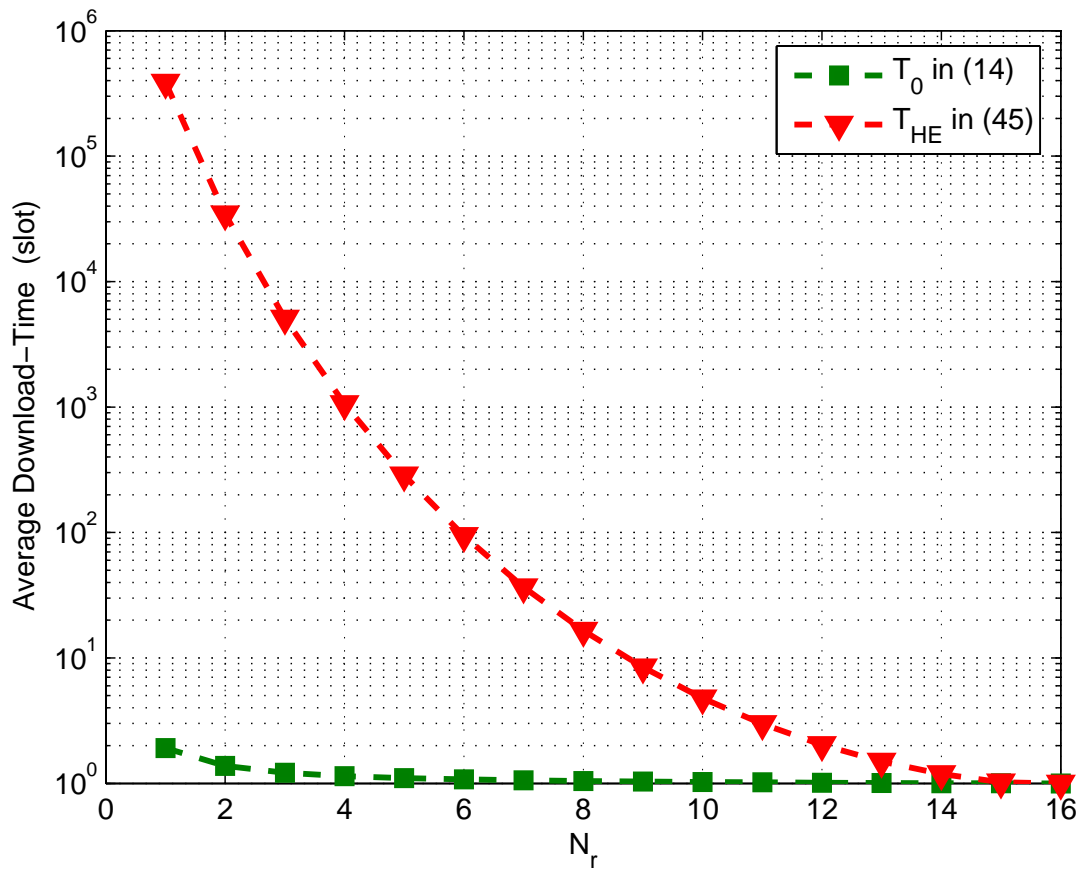


Fig. 4. Average download-times for the optimal and heuristic policies versus the number N_r of receive antennas for a SIMO-MRC system with $\mathcal{N}_0 = 1(\text{Joule})$, $\mathcal{E}_{max} = 10(\text{Joule})$, $K = 2(\text{Joule/nat})$ and $\Delta = 100(\text{Mnat})$.

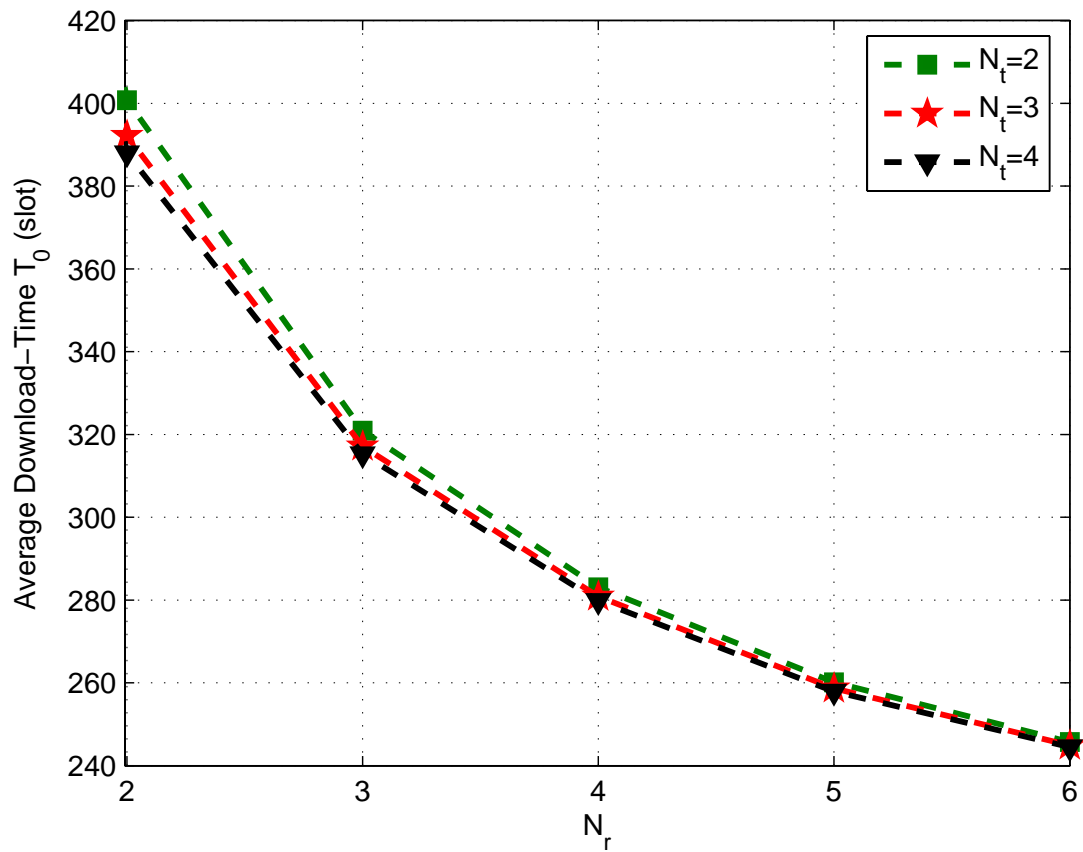


Fig. 5. Average download-times $T_0(\text{slot})$ of the optimal policy (36) for MIMO systems equipped with unit-rate OSTBCs. The system parameters are $N_0 = 1(\text{Joule})$, $E_{max} = 10(\text{Joule})$, $K = 2.5(\text{Joule/nat})$, and $\Delta = 100(\text{Mnats})$.

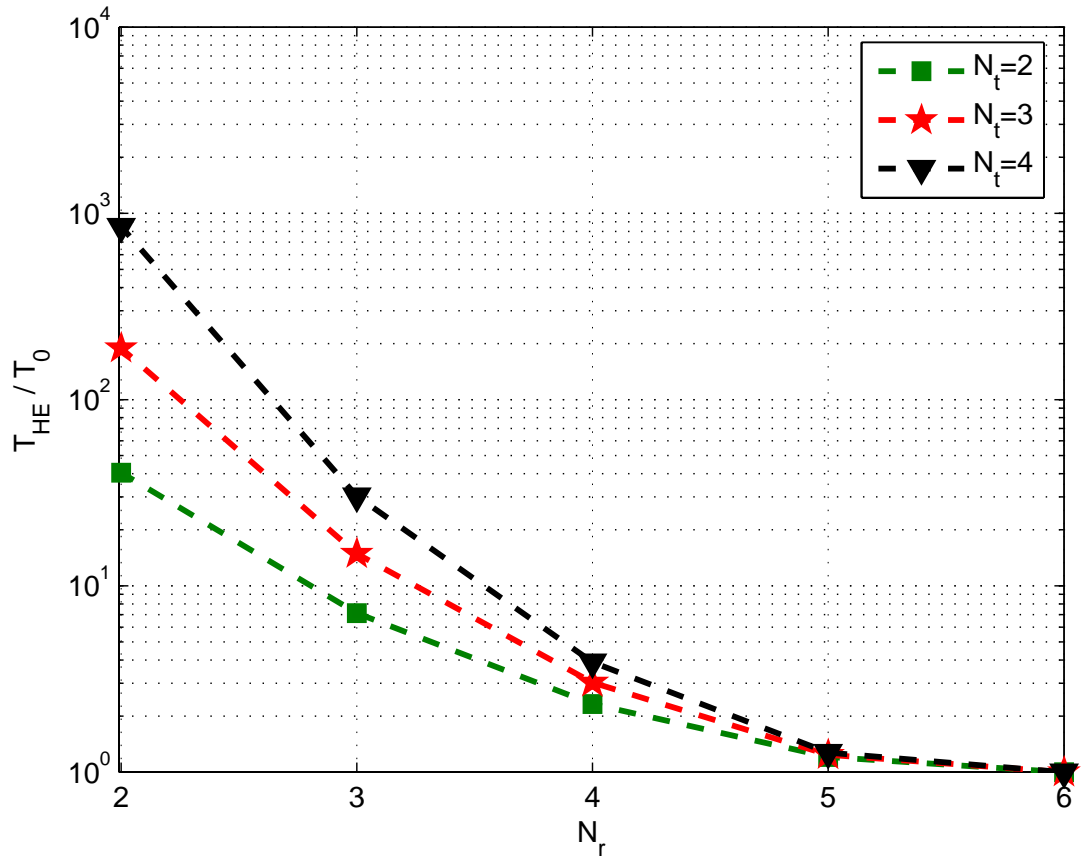


Fig. 6. T_{HE}/T_0 ratio among the average download-times of the heuristic and optimal policies for the same MIMO systems considered in Fig.5