A New Family of Decision Delay-Constrained MAP Decoders for Data Transmission over Noisy Channels with ISI and Soft-Decision Demodulation

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Abstract - A new family of nonlinear decision delay-constrained receivers minimizing the symbol-decoding-error probability of QAM- or PSK-modulated digital information sequences transmitted over time-dispersive time-varying noisy waveform channels is presented. New (generally) time-varying Bhattacharyya-type upper bounds for the performance evaluation of the proposed receivers are also presented.

Summary

In this work a novel solution for the optimal synthesis of nonlinear receivers for the detection of digitally modulated QAM or PSK information sequences transmitted over time-varying channels impaired by known ISI and additive noise is presented for the case when the decoding-decision delay \( \Delta \) is limited and finite. Receivers which minimize the symbol error probability (i.e., symbol-by-symbol MAP decoders) are considered.

The known solution presented in [1] for a similar problem holds for the case of data transmission systems with time-invariant waveform channels and unquantized soft-decision demodulation. Moreover, the algorithm in [1] has been obtained by means of a direct application of Bayes’ rule so that the resulting receiver complexity grows exponentially with the decision-delay \( \Delta \). As a consequence, the implementation of such receivers for multilevel digital signaling seems to be unattractive even for small values of \( \Delta \) [4, Sect.6.6].

In this work a M-level quantizer is assumed present at the output of the noisy waveform channel so that the finite word-length effects of digital receivers can be suitably taken into account. Moreover, the approach followed to synthesize the MAP decoder is completely different from that in [1] and is based on the modeling of the ISI channel as a sequential Moore-type finite-state-machine. This allows to adopt the recursive Kalman-like algorithms of [2] for the computation of the sequence \( \{n(k \mid k+\Delta)\} \), where \( n \) is the noise term generated by an A Posteriori Probabilities (APPs) of the so-called "channel state" Markov chain \( \{x(k)\} \in \mathcal{U} = \{u_1, u_2, \ldots, u_K\} \) for every assigned decision-delay \( \Delta \). The main advantage of this approach is that the implementation of the resulting MAP decoders exhibits a complexity which grows only linearly with the value assumed by the decision-delay \( \Delta \). In fact, the following expression for the computation of the APP sequence (recursive with respect to the k-index) holds:

\[
\begin{align*}
\pi(k+1 \mid k+\Delta) = & \pi(k \mid k+\Delta-1) + \\
& \sum_{m=1}^{K+1} \pi(k,m) \pi(m),
\end{align*}
\]

(1)

In (1) the matrix \( \pi \) is the probability transition matrix of the Markov chain \( \{x(k)\} \) and the sequences \( \{\pi(k,m)\} \) and \( \{\pi(m)\} \) can be recursively calculated as in [2]. Starting from the above APPs sequence, the corresponding MAP estimate sequence \( \{\hat{x}_{\text{MAP}}(k) \mid k+\Delta \in \mathcal{A}\} \) of the transmitted S-ary information sequence \( \{a(k)\} \in \mathcal{A} = \{a_1, a_2, \ldots, a_S\} \), \( k \geq 0 \) can be easily computed following quite standard procedures (see, for example, [3, Sec.VI]).

As far as the performance evaluation of the mentioned nonlinear MAP decoders is concerned we observe that, from the authors' knowledge, no explicit analytical expressions are known in literature (see [4, Sect.6.6]). Starting from Eq.(1), new (generally) time-varying Bhattacharyya-type upper bounds for the performance evaluation of the proposed MAP decoders have been derived as follows:

\[
\begin{align*}
P(\hat{x}_{\text{MAP}}(k) \mid k+\Delta) & \leq \\
& \sum_{m=1}^{M+1} \sum_{s=1}^{S} \sum_{n=1}^{N_{\text{MAP}}} \left[ \left( y_{m+1}^{\Delta} - y_{m}^{\Delta} \right)^{2} \pi(a(k) = a_{n}) \pi(a(k) = a_{n}) \right],
\end{align*}
\]

(2)

where \( y_{m+1}^{\Delta}(m) \) is the m-th determination assumed by the ordered random sequence \( Y_{m+1}^{\Delta} \), constituted by the quantized noisy data received at the channel output from step 0 to step \( k+\Delta \). Simulation results proved that the upper bounds of Eq.(2) are quite tight for error probabilities below 10^{-2}.

The performance of the proposed symbol-by-symbol MAP receivers have been compared to that pertaining to the conventional sequence Maximum Likelihood (ML) receivers (based on the classic Viterbi Algorithm with optimized branch metric). Computer simulations showed that the performance of the presented receivers overcomes that of the ML sequence receivers when the transmitted channel is largely time-dispersive and the signal-to-noise ratio (SNR) at the receiver site is quite low, so that the proposed decoders could be attractive, in particular, for HF channel equalization. Moreover, for the MAP decoders at hand a decision-delay \( \Delta \) of the order of the length \( L \) of the channel impulse response (measured as multiples of the signaling period \( T \) ) results in a negligible performance loss with respect to the ideal case \( \Delta = \infty \), while a delay \( \Delta \) of 5-6 times the length \( L \) is in general required for the corresponding ML decoders.

As illustrative example, in Table I the bit-error-rate (BER) for the case of a BPSK-modulated binary message sequence crossing the discrete-time baseband ISI channel of [4], Tab.6.7.1, of length \( L=6 \) are reported. Hard-decision demodulation and AWGN are assumed; the signal-to-noise ratio is evaluated at the input of the receiver quantizer. In Tab.II the corresponding steady-state values of the Bhattacharyya-like bound (2) are reported. In [5] the symbol-by-symbol MAP decoders described in this work are employed for decoding Trellis-encoded data sequences. It is finally observed that if the transmitted sequences are equiprobable, the proposed MAP receivers coincide with the corresponding symbol-by-symbol ML receivers.

**REFERENCES**


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<th>Channel of length ( L=6 )</th>
<th>Sequence detection (%)</th>
<th>Proposed detectors</th>
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<tr>
<td>( \Delta = 0 )</td>
<td>( \Delta = L-1 )</td>
<td>( \Delta = \infty )</td>
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<tr>
<td>SNR = 7</td>
<td>0.2822</td>
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<th>Upper bounds on BER</th>
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<th>( \Delta = L-1 )</th>
<th>( \Delta = \infty )</th>
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<td>SNR = 15</td>
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| Tab.I | Tab.II |