Mars surface parameters estimation
A surface in the (x, y) plane is described using a bi-dimensional function \( z(x, y) \) of height, with proper statistical properties.

The main parameters which tell us about the statistical variability of the surface are:

- **Standard deviation of surface height**: \( \sigma_z \)

For a statistically representative segment of the surface, of dimension \( L_x \) and \( L_y \), the mean height of the reference surface is:

\[
\overline{z} = \frac{1}{L_x L_y} \int_{-L_x/2}^{L_x/2} \int_{-L_y/2}^{L_y/2} z(x, y) \, dx \, dy
\]

The autocorrelation function for a one-dimensional surface profile \( z(x) \) is defined as

\[
\overline{z^2} = \frac{1}{L_x L_y} \int_{-L_x/2}^{L_x/2} \int_{-L_y/2}^{L_y/2} z^2(x, y) \, dx \, dy
\]

\[
\sigma_z = \sqrt{\overline{z^2} - \overline{z}^2}
\]
The function is measuring the similarity existing between the heights of two points distant ‘x’ one from the other. The surface correlation length is defined as the distance ‘Lc’ for which:
\[ \rho(Lc) = \frac{1}{e} \]
We will assume two heights uncorrelated when their horizontal distance is greater than Lc.
It’s usual to define a third parameter:
• **rms slope m**

\[ m_s = \sqrt{S^2} \]

where we indicate with ‘S’ the slope of z (x₀) in the point (x₀).
Surface description

\[ S = \lim_{\Delta x \to 0} \frac{z(x_0 + \Delta x) - z(x_0)}{\Delta x} \]

the rms slope is the standard deviation of the local slope of the surface

The surfaces can be defined through:

1. a gaussian probability distribution of heights:

\[ p(z) = \frac{1}{\sqrt{2\pi\sigma_h}} e^{-\frac{z^2}{2\sigma_h^2}} \]

2. an autocorrelation function to be selected among the gaussian or exponential distributions (\( L_c \) is the correlation length):

\[ \rho(x) = e^{-\left( \frac{x}{L_c} \right)^2} \]

\[ m_s = \sqrt{2} \cdot \frac{\sigma_h}{L_c} \]
Surface description

MARS Surface was described by:

- **large scale scattering contribution**, results from gentle geometrical undulations of the surface on a scale of many hundreds to thousands meters;

- **small scale scattering contribution**, gives reason of the fast and slight variations of the surface height over an horizontal scale of some tenths of meters.

<table>
<thead>
<tr>
<th>Large Scale Component</th>
<th>Small Scale Component</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rms slope</strong> $(m_{s1})$</td>
<td><strong>Rms Slope</strong> $(m_{s2})$</td>
</tr>
<tr>
<td>0.005 - 0.1 rad.</td>
<td>0.1 – 0.6 rad. $(5.7° - 34.4°)$</td>
</tr>
<tr>
<td><strong>Correlation Length</strong> $(L_1)$</td>
<td><strong>Rms height</strong> $(\sigma_{h2})$</td>
</tr>
<tr>
<td>100 – 30000 m</td>
<td>0.1 – 1 m</td>
</tr>
</tbody>
</table>
Surface description

Correlation function: The correlation function of a surface is related to the Allan variance through the following relation [1]:

\[ \nu^2(\Delta x, \Delta y) = \left\langle \left( (z(x, y) - z(x + \Delta x, y + \Delta y))^2 \right) \right\rangle = 2\left[ \sigma_h^2 - \text{Cov}(\Delta x, \Delta y) \right] \] for stationary functions

\[ \Delta x \to 0 \Rightarrow \text{identical profiles} \quad \Delta x \to \infty \Rightarrow \text{uncorrelated profiles} \Rightarrow \nu^2 = 2\sigma_h^2 \]

\[ \rho(\Delta x, \Delta y) = \frac{\text{Cov}(\Delta x, \Delta y)}{\sigma_h^2} \Rightarrow \nu^2(\Delta x, \Delta y) = 2\sigma_h^2(1 - \rho(\Delta x, \Delta y)) \]

With a substitution in the previous relation and taking into account that [1]

\[ \nu^2(\Delta x) = c^2 \cdot \Delta x^{2H} \]

Where H is the Hurst coefficient we obtain

\[ \rho(\Delta x) = 1 - \frac{c^2}{2\sigma_h^2} |\Delta x|^{2H} \]

RMS slope

\[ s(\Delta x) = \frac{\nu(\Delta x)}{\Delta x} \quad \text{for a self-affine profile:} \quad s(\Delta x) = v_0 \frac{\Delta x^{H-1}}{\Delta x_0^{H-1}} = s(\Delta x_0) \left( \frac{\Delta x}{\Delta x_0} \right)^{H-1} \]

Surface description

According to the large-scale morphology and by considering a surface with an autocorrelation function $\rho(x,y)$ it can be written:

- **rms height**
  \[ \sigma^2(L) = \sigma_z^2(1 - \rho(L)) \]
  \[ \rho(L) \rightarrow \frac{1}{L^2} \int \int L \rho(x,y) dx dy \]

- **Allan deviation (L>>Δx)**
  \[ \overline{v(\Delta x)}^2 = \frac{1}{L} \sum_{L_k} \frac{1}{N} \sum_j (z(x_j) - z(x_j + \Delta x))^2 = \frac{1}{L} \sum_{L_k} \frac{2}{N} \left( \sum_j z(x_j)^2 - \sum_j z(x_j)z(x_j + \Delta x) \right) \rightarrow 2\sigma_z^2(1 - \rho(\Delta x)) \]

  \[ \rho(\Delta x) = 1 - c^2 \frac{|\Delta x|^{2H}}{K\sigma^2} \]

- **rms slope**
  \[ \overline{s(\Delta x)} = \sqrt{2} \frac{\sigma_z \sqrt{(1 - \rho(\Delta x))}}{\Delta x} \rightarrow \sqrt{2} \frac{\sigma_z c |\Delta x|^{H-1}}{\sqrt{K\sigma}} \]
Surface available data

- Mars surface point to point slope exhibits a power law behavior vs the lag distance.
- As can be seen in the next slide some profile verifies a power law at scales from 300 m up to 10 km while many segments departs from fractal behavior at lag distance less than 1 km.
- A best fitting of the experimental data has been obtained with the stationary Gaussian amplitude model correlated with exponential correlation function.
Surface available data

Given the surface data and assuming the following correlation function it is possible to show the matching between the available estimated data and the theoretical expression

\[ \rho(\Delta x) = \exp\left(-\frac{|\Delta x|^{2H}}{\ell^{2H}}\right) \]

Correlation function with \( \ell \) correlation length:

rms slope:

\[ s(\Delta x) = \sqrt{2} \left( 1 - \exp\left(-\frac{|\Delta x|^{2H}}{\ell^{2H}}\right) \right) \]

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In conclusion it can be assumed that the following values cover almost cases of Mars surface:

\[ H = 0.6 - 1, \quad s(100) = 10^{-2} - 10^{-1}, \quad l = 3 - 12 \text{ Km} \]

H, s(100) and l appear uncorrelated
MARSIS surface characteristics

In the following two slides are shown the statistical parameters relevant to Mars south pole.
In particular the next 2 slides synthesize the average values and the standard deviation for several parameters (e.g. rms height, correlation length, hurst, etc.) while the other slides show the complete probability density function. The Mars surface statistical characteristics are Global (averaged on 50 km·50 km corresponding to 5 km exploration depth) and Local when referred to the local facet and averaged on 10 km·10 km).
# MARSIS

**Surface statistical characteristics**

Global (evaluation of the statistical parameters for all points in a circular area (radius 50 km)

The first row is relevant to Longitude $-180^\circ$ to $0^\circ$, the second row to Longitude $0^\circ$ to $180^\circ$.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum value</th>
<th>Maximum value</th>
<th>Average value</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_{\mu} = \sqrt{\sigma_r^2 + (\sigma_{\mu_a}^2 + \sigma_{\mu_o}^2) \cdot 25 \cdot 10^6}$ m</td>
<td>3.1</td>
<td>5.1</td>
<td>103.9</td>
<td>72.8</td>
</tr>
<tr>
<td>$G_{\text{SIGMA}_h}$</td>
<td>354.8</td>
<td>367.3</td>
<td>111.8</td>
<td>60.8</td>
</tr>
<tr>
<td>$\bar{L}<em>i = \frac{1}{n} \sum</em>{j=1}^{n} \left( \frac{1}{L_{ij}} + \frac{1}{L_{jej}} \right)$</td>
<td>1061</td>
<td>1503</td>
<td>2294.8</td>
<td>280.7</td>
</tr>
<tr>
<td>$G_{\text{CL INV}}$</td>
<td>2915</td>
<td>2964</td>
<td>2388.6</td>
<td>305.8</td>
</tr>
<tr>
<td>$\bar{H}<em>i = \frac{1}{n} \sum</em>{j=1}^{n} H_j$</td>
<td>0.41</td>
<td>0.47</td>
<td>0.81</td>
<td>0.1</td>
</tr>
<tr>
<td>$G_{\text{H}}$</td>
<td>0.94</td>
<td>0.95</td>
<td>0.83</td>
<td>0.057</td>
</tr>
<tr>
<td>$\bar{m}_i = \sqrt{\frac{2}{L_i}} \sigma_r$</td>
<td>0.003</td>
<td>0.004</td>
<td>0.063</td>
<td>0.044</td>
</tr>
<tr>
<td>$G_{\text{SLOPE}}$</td>
<td>0.213</td>
<td>0.204</td>
<td>0.07</td>
<td>0.037</td>
</tr>
<tr>
<td>$\bar{a}_i (\lambda) = \sqrt{\frac{2}{L_i} \cdot \frac{\lambda^2}{\pi}}$</td>
<td>0.011</td>
<td>0.013</td>
<td>0.094</td>
<td>0.057</td>
</tr>
<tr>
<td>$G_{\text{SLAMBDA}}$</td>
<td>0.302</td>
<td>0.283</td>
<td>0.11</td>
<td>0.051</td>
</tr>
<tr>
<td>$\Delta \sigma_i</td>
<td>_{\text{max}} = (\sigma_i</td>
<td>_{\text{max}} - \sigma_i)$</td>
<td>2.4</td>
<td>14.1</td>
</tr>
<tr>
<td>$k_{\sigma_r} \cdot \sigma_\tau \rightarrow k_\sigma$</td>
<td>2.3</td>
<td>12.9</td>
<td>4.7</td>
<td>1.17</td>
</tr>
<tr>
<td>$\Delta \alpha_{xi}</td>
<td><em>{\text{max}} = (\alpha</em>{xi}</td>
<td><em>{\text{max}} - \alpha</em>{xi})$</td>
<td>0.9</td>
<td>13.8</td>
</tr>
<tr>
<td>$k_{ax} \cdot \sigma_{xi \alpha \alpha} \rightarrow k_{ax}$</td>
<td>0.7</td>
<td>11.6</td>
<td>4.02</td>
<td>1.38</td>
</tr>
<tr>
<td>$\Delta \alpha_{yi}</td>
<td><em>{\text{max}} = (\alpha</em>{yi}</td>
<td><em>{\text{max}} - \alpha</em>{yi})$</td>
<td>0.8</td>
<td>15.1</td>
</tr>
<tr>
<td>$k_{ay} \cdot \sigma_{yi \alpha} \rightarrow k_{ay}$</td>
<td>0.7</td>
<td>11.1</td>
<td>4.19</td>
<td>1.41</td>
</tr>
</tbody>
</table>
**MARSIS** surface statistical characteristics

Local (evaluation of the statistical parameters every 5 km on a facet of 10 km by 10 km
The first row is relevant to Longitude $-180^\circ$-$0^\circ$, the second row to Longitude $0^\circ$-$180^\circ$

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Minimum value</th>
<th>Maximum value</th>
<th>Average value</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_i$</td>
<td>0.5</td>
<td>350</td>
<td>29.2</td>
<td>34.4</td>
</tr>
<tr>
<td>$L_{\text{SIGMA}_h}$</td>
<td>0.5</td>
<td>306.5</td>
<td>28</td>
<td>32.7</td>
</tr>
<tr>
<td>$L_i = \frac{2}{\left(\frac{1}{L_{xi}} + \frac{1}{L_{yi}}\right)}$</td>
<td>230.6</td>
<td>4479.5</td>
<td>2296.16</td>
<td>583</td>
</tr>
<tr>
<td>$L_{\text{CL INV}}$</td>
<td>333</td>
<td>4289</td>
<td>2388</td>
<td>553.5</td>
</tr>
<tr>
<td>$H_i = \frac{H_{xi} + H_{yi}}{2}$</td>
<td>0.202</td>
<td>1.0</td>
<td>0.81</td>
<td>0.15</td>
</tr>
<tr>
<td>$L_H$</td>
<td>0.252</td>
<td>1.245 ?</td>
<td>0.83</td>
<td>0.12</td>
</tr>
<tr>
<td>$L_{\text{SLOPE}}$</td>
<td>0.0003</td>
<td>1</td>
<td>0.018</td>
<td>0.02</td>
</tr>
<tr>
<td></td>
<td>0.0003</td>
<td>0.27</td>
<td>0.02</td>
<td>0.02</td>
</tr>
<tr>
<td>$L_{\text{SLAMBDA 166}}$</td>
<td>0.002</td>
<td>1.5</td>
<td>0.024</td>
<td>0.028</td>
</tr>
<tr>
<td></td>
<td>0.002</td>
<td>0.4</td>
<td>0.024</td>
<td>0.023</td>
</tr>
<tr>
<td>$z_i$</td>
<td>201</td>
<td>4810</td>
<td>2131</td>
<td>681.5</td>
</tr>
<tr>
<td>$L_Z$</td>
<td>-160</td>
<td>4882</td>
<td>2182</td>
<td>782.4</td>
</tr>
<tr>
<td>$\alpha_{xi}$</td>
<td>-7.78</td>
<td>9.36</td>
<td>0.13</td>
<td>0.95</td>
</tr>
<tr>
<td>$L_{\text{alfax}}$</td>
<td>-11.62</td>
<td>10.44</td>
<td>0.13</td>
<td>1.09</td>
</tr>
<tr>
<td>$\alpha_{yi}$</td>
<td>-8.91</td>
<td>8.67</td>
<td>-0.001</td>
<td>1.05</td>
</tr>
<tr>
<td>$L_{\text{alfay}}$</td>
<td>-9.29</td>
<td>8.8</td>
<td>0.003</td>
<td>0.16</td>
</tr>
</tbody>
</table>
Mars surface parameters estimation

- The following procedure has been utilized to obtain the surface statistical parameters by MOLA data:
  - Selection of the 10*10 km² regions, overlapped of 5 Km
  - Flat surface estimation for each region by best fitting. With reference to the flat surface for each region must be given:
    - $z_i$ mean value of the surface height
    - $\alpha_{xi}, \alpha_{yi}$ surface slopes in x and y direction and
  - With reference to the difference between input MOLA data and the flat surface for each region must be evaluated:
    - $\sigma_i$ rms surface
    - $L_{xi}, L_{yi}$ correlation length
    - $H_i$ Hurst coefficient
Mars surface parameters estimation
Mars surface parameters estimation
Mars surface parameters estimation
Mars surface parameters estimation
Mars surface parameters estimation
Mars surface parameters estimation
Mars surface characteristic $\sigma_h$
Mars surface characteristic $L_i$
Mars surface characteristic H

LONG: (-180, 0)  LAT: (-88, -77)  dx= ~ dy = ~10 km
min = 0.41  max = 0.94  m = 0.813316  dev = 0.103353
Mars surface characteristic $k_{\alpha x}$
Mars surface characteristic $k_{\alpha y}$

LONG: (-180, 0)  LAT: (-88, -77)  dx~dy~10 Km
min = 0.8  max = 15.1  m = 4.41981  devs=1.59258

PDF

$G_{K3\_alfay}$
Mars surface characteristic

Local
Mars surface characteristic $\sigma_h$
Mars surface characteristic $L_i$
Mars surface characteristic H

LONG: (-180, 0)  LAT: (-88, -77)  dx=~  dy= ~5 km
min= 0.202  max= 1.056  m= 0.813786  devs=0.151285
Mars surface characteristic $s(\lambda)$
Mars surface characteristic $\alpha_x$
Mars surface characteristic $\alpha_y$
Slope map