Pulse Shaping vs. OFDM Transmission Techniques for InHome Power Line Communications

Mauro Biagi, Simone Greco, Stefano Rinauro and Roberto Cusani
Dept. of Information, Electronics and Telecommunication (DIET) eng., University of Rome Sapienza, Rome, Italy, Email: {Mauro.Biagi, Simone.Greco, Stefano.Rinauro, Roberto.Cusani}@uniroma1.it

I. INTRODUCTION

The nature of the transmission medium (the electrical cable) and possible power/interference constraints coming from coexistence issues, forces Power Line Communication (PLC) signals to respect some emission limits [1, Chap.5]. Within this framework, pulse shaping may offer effective solutions to maximize Signal-to-Interference-Noise Ratio (SINR), while abiding by spectral constraints. The model of the received signal is given by

\[ y(t) = s(t) \otimes h(t) + n(t) + d(t) \]  

(1)

where \( s(t) \) is the PLC signal, \( h(t) \) represents channel impulse response, \( \otimes \) is the convolution operator, \( n(t) \) is zero mean, \( N_0 \)-variance Additive White Gaussian Noise (AWGN), \( d(t) \) accounts for possible in-band disturbing interference due to coexistence issues. Several parameters like cable gauge and quality, bandwidth and network topology influence overall channel characteristics. The spectrum \( S(\omega) \) of the PLC signal should match CENELEC/ETSI spectral masks [2, Chap.7] and at the same time should allow to attain the required system performance. Several transmission techniques have been proposed in the last years for this purpose. OFDM offers effective solutions (see e.g. [3] and [4]) but also exhibits some drawbacks, in particular:

- high sensitivity inter-channel interference (ICI)
- sensitivity to frequency, clock and phase offset
- relatively large peak-to-average ratio which reduces the power efficiency of the RF amplifier (non-linear amplification destroys the orthogonality of the OFDM signal and introduces out-of-band spectral components).

II. SPECTRAL APPROACH

The conventional approach for spectral mask matching is filtering (low-pass, band-pass). In this paper, we aim at realizing a spectral pulse shape \( S(\omega) \), which closely follows the mask \( S_{\text{mask}}(\omega) \) by minimizing the weighted error

\[ E(\omega) = W(\omega) | S_{\text{mask}}(\omega) - S(\omega) | , \]  

(2)

where \( W(\omega) \) represents the weight vector for the error at different frequencies.

Parks-McClellan algorithm [5] tries to solve the above minimization problem by considering cosinusoidal polynomial terms:

\[ S(\omega) = \sum_{m=0}^{M} a(m) \cos(\omega m) = \]  

\[ = \sum_{k=0}^{M} a(k) (\cos(\omega))^k = P(x) \bigg|_{x=\cos(\omega)} \]  

(3)

where the term \( P(x) \) indicates the polynomial evaluated in the cosinusoidal terms. Now, affording (2) as a min-max Chebyshev problem, we obtain

\[ \min_{\{a(k)\}} \left[ \max_{\omega \in \Gamma} |E(\omega)| \right] = \]  

\[ = \min_{\{a(k)\}} \left[ \max_{\omega \in \Gamma} W(\omega) \left( S_{\text{mask}}(\omega) - \sum_{k=0}^{M} a(k) \cos(\omega k) \right) \right] \]  

(4)

where \( \Gamma \) is the interval \((0, \pi]\) wherein the optimization is performed.

A. Pulse-Shaping for PLC

The problem can be stated in the following way

\[ W(\omega_i) | S_{\text{mask}}(\omega_i) - P(\omega_i) | = (-1)^n , \]  

(5)

where \( n \) is the maximum tolerated approximation error for the frequencies \( \omega_i \) and \((-1)^n\) accounts for positive and negative error ripples [5]. From the last expression, and considering possible different values for the errors, we can write

\[ \sum_{k=0}^{M} a(k) \cos(\omega_i k) + \frac{(-1)^{n-1}}{W(\omega_i)} = S_{\text{mask}}(\omega_i) . \]  

(6)

In order to supply an approximation-tuning mechanism for each one of the \( M+1 \) considered points (which define the pulse order), we consider possibly different error levels \( n \) at different frequencies.

The conventional mask matching or filter design take care only of emission limits or, analogously, the ripple specifications coming from attenuation/enhancement issues. Here, the role played by the effect of noise (or noise plus interference) suggests to try to mitigate (or suppress) the effect of interference/noise by zeroing (or attenuating) the spectral components where disturbing signals are more strong.

This aspect directly reflects on the max approximation errors \( n \) which grow when the interference level grow up.
III. NUMERICAL RESULTS

In Fig. 1, a power budget of $P=1$ Watt and emission limits according to [2, Chap.7] are assumed to compare the allotted power on a bandwidth of 29Mhz (from 1Mhz to 30Mhz). The green plot represents the bound coming from EMC issues. The blue and star-marked plot refers to noise shape while the black and magenta curves are related to pulse shaping. More in detail the black and diamond-marked line depicts the performance of the pulse shaping when the channel is not considered in the design procedure and only the noise and mas are considered. As soon as channel features are taken into account (see magenta circle-marked line) for some frequencies (i.e. between 18 and 19Mhz) the approximation error is larger since the noise component is high and the channel attenuation becomes more evident. In order to compare pulse shaping with bit-loading procedure as that described in [3] we can consider different performance metrics. First of all, let us refer to the Peak to Average Power Ratio (PAPR). When transmitting data, a power amplifier is required to boost the outgoing signal to a level high enough to assure the link reliability. The power amplifier is one of the biggest consumers of energy and should thus be as power efficient as possible to increase the operation time of the device on a battery charge. The efficiency of a power amplifier depends on two factors that are the ability of amplifying the highest peak value of the wave and the requirement of not transporting by the peaks more information than the average power of the signal over time. The PAPR is formally defined as

$$\text{PAPR} = \frac{|x|_{\text{peak}}}{x_{\text{rms}}}$$  \hspace{1cm} (7)\]

and when OFDM is used the value achieved is 12dB while in the pulse shaping case we have an average value of 8.5dB (std dev of 2dB).

Dealing with clock off-set (synchronization) it is important to compare the pulse-shaping procedure with OFDM when channel estimation is affected by errors. The results in Table I show that OFDM outperforms pulse shaping (when a 200Mb/s rate has to be guaranteed) since when perfect synchronization is considered, OFDM presents a BER of $1.18 \times 10^{-6}$ while pulse-shaping raises to $2 \times 10^{-5}$. When the percentage synchronization error increases $\eta = T_s/T_b$ (being $T_s$ the synchronization error and $T_b$ the pulse length), the performance of pulse shaping are better than those of OFDM. In order to summarize strenght point of the above techniques and their drawbacks, let us make reference to Table II. it is possible to argue that OFDM is better than Pulse-Shaping when BER must be minimized subject to channel capacity constraints. Furthermore, since OFDM frequency partitioning is decided once for all and before knowing channel and noise features while in the pulse shaping procedure the order of the filter is strongly dependent on channel features, the complexity of OFDM is lower and this technique is InterSymbol Interference (ISI) resistant. Both the approaches are able to be Multi-User Interference (MUI) resilient while pulse shaping does not present Inter Carrier Interference (ICI), it is synchronization robust and present lower PAPR. As a conclusive remark we can argue that no techniques able to gain in each field exist. The only one point to investigate is the possibility to merge the strength points of these approaches in a common hybrid framework.

### TABLE I

<table>
<thead>
<tr>
<th>Percentage synchronization error</th>
<th>BER OFDM</th>
<th>BER Pulse Shaping</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$1.18 \times 10^{-6}$</td>
<td>$2 \times 10^{-5}$</td>
</tr>
<tr>
<td>1</td>
<td>$4.56 \times 10^{-6}$</td>
<td>$4.53 \times 10^{-5}$</td>
</tr>
<tr>
<td>2</td>
<td>$1.32 \times 10^{-5}$</td>
<td>$7.94 \times 10^{-5}$</td>
</tr>
<tr>
<td>5</td>
<td>$6.88 \times 10^{-6}$</td>
<td>$1.11 \times 10^{-4}$</td>
</tr>
<tr>
<td>10</td>
<td>$2.21 \times 10^{-4}$</td>
<td>$2.13 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

### TABLE II

<table>
<thead>
<tr>
<th></th>
<th>OFDM</th>
<th>Pulse Shaping</th>
</tr>
</thead>
<tbody>
<tr>
<td>PAPR</td>
<td>$\times$</td>
<td>$\checkmark$</td>
</tr>
<tr>
<td>Synchronization</td>
<td>$\checkmark$</td>
<td>$\times$</td>
</tr>
<tr>
<td>BER</td>
<td>$\checkmark$</td>
<td>$\times$</td>
</tr>
<tr>
<td>MUI</td>
<td>$\checkmark$</td>
<td>$\times$</td>
</tr>
<tr>
<td>Complexity</td>
<td>$\checkmark$</td>
<td>$\times$</td>
</tr>
<tr>
<td>ISI</td>
<td>$\checkmark$</td>
<td>$\times$</td>
</tr>
</tbody>
</table>

### REFERENCES